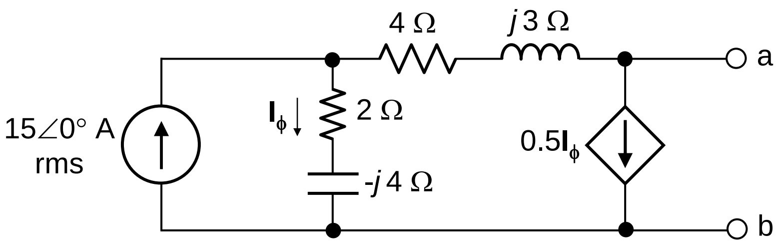
Problem (12 pts)

Consider the circuit shown



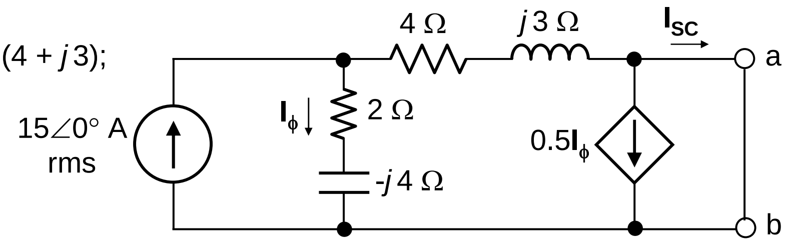
a. Determine the open-circuit voltage at terminals a and b. (3 pts)

$15 = 1.5\mathbf{I}_\phi$; $\mathbf{I}_\phi = 10$ A; $\mathbf{V}_{Th} = 10(2 - j4) - 5(4 \boxed{+} j3) = -j55$ V.

b. Determine the current flowing in the short circuit when there is a short between terminals a and b. (3 pts)

$15 = 1.5\mathbf{I}_\phi + \mathbf{I}_{SC}$;
$\mathbf{I}_\phi(2 - j4) = (0.5\mathbf{I}_\phi + \mathbf{I}_{SC})(4 + j3)$;
solving these two equations gives

$$\mathbf{I}_{SC} = \frac{165}{74}(1 - j6) \text{ A}$$



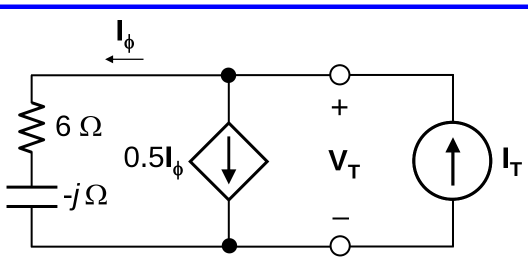
c. Determine the equivalent impedance Z_{Th} as seen by the terminals a and b. (2 pts)

$$Z_{Th} = \cancel{|\mathbf{V}_{Th}|/|\mathbf{I}_{SC}|} = \frac{\boxed{-}j55 \times 74}{165(1 - j6)} = 4 - j\frac{2}{3}\,\Omega$$

d. Evaluate Z_{Th} again using a different method then that employed in part (c). (4 pts)

$\mathbf{I}_T = 1.5\mathbf{I}_\phi$; $\mathbf{V}_T = \mathbf{I}_\phi(6 - j)$
dividing gives:

$$Z_{Th} = \frac{6 - j}{1.5} = 4 - j\frac{2}{3}\ \Omega.$$



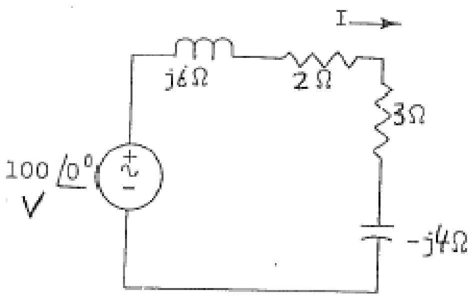


Figure 4.

4. Find the current in the circuit shown in figure 4.

A. 18.6 $\angle -21.8°$ A
B. 22.5 $\angle -35.6°$ A
C. 12.3 $\angle -18.9°$ A
D. 34.7 $\angle -29.7°$ A
E. None of the above

Hint: redraw lattice circuit as a bridge

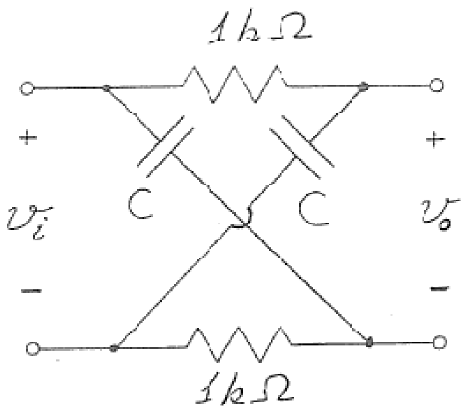


Figure 9.

9. Determine C in the circuit shown in figure 9 so that the output voltage v_o has the same magnitude as the input voltage v_i but lags it by 90°, assuming ω = 200 rad/s.

A. 5 μF
B. 2 μF
C. 6 μF
D. 8 μF
E. None of the above

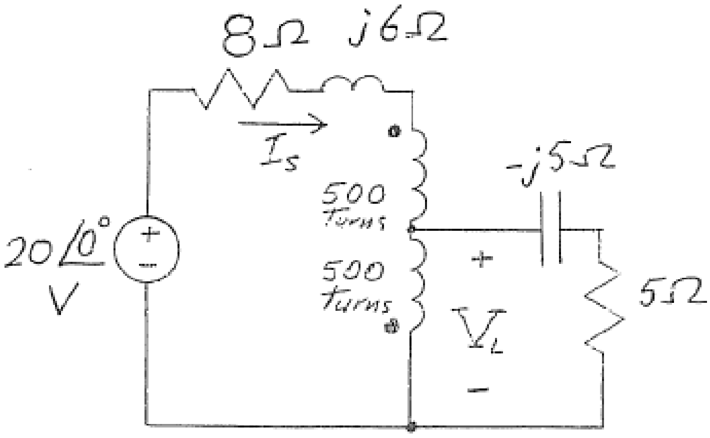


Figure 12.

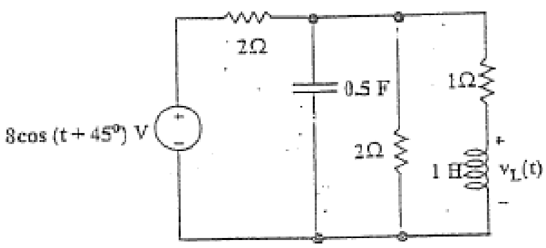
12. Determine I_s and V_L in the circuit shown in figure 12.

A. $1.4\angle -45.0°$ A, 0 V
B. $0.7\angle -45.0°$ A, $0.3\angle 45.0°$ V
C. $1.4\angle -36.3°$ A, $0.3\angle 14.4°$ V
D. $1.1\angle -45.0°$ A, $0.4\angle 45.0°$ V
E. None of the above — $2\angle -36.9$ A, 0 V

Hint: determine current in (5 - j5) ohms, assuming autotransformer is ideal

4. Find the expression of $v_L(t)$ in the circuit shown in Fig. 3.

A. $v_L(t) = 1.89\cos(t + 90°)$ V
B. $v_L(t) = 1.24\cos(t - 90°)$ V
C. $v_L(t) = 2.58\cos(t + 45°)$ V
D. $v_L(t) = 0.96\cos(t - 45°)$ V
E. None of the above



6. Find v_o in the circuit shown in Fig. 5 if $\omega = 5 \times 10^6$ rad/s.

A. 14.7 $\angle 21.8°$ V
B. 11.6 $\angle 15.6°$ V
C. 12.8 $\angle 35.2°$ V
D. 10.5 $\angle 25.9°$ V
E. None of the above

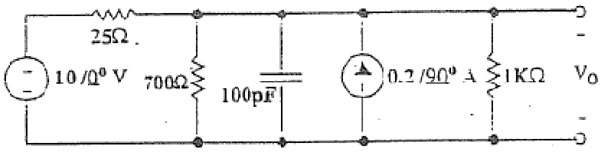
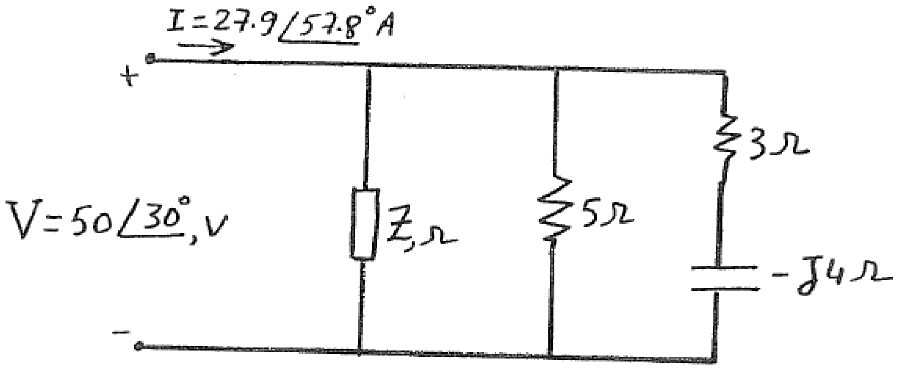


Figure 5.

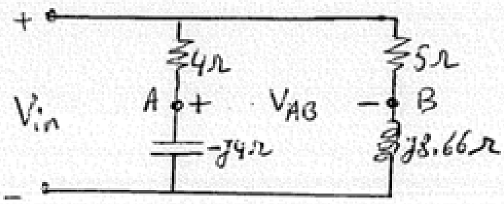
12. Determine Z in the circuit shown below:



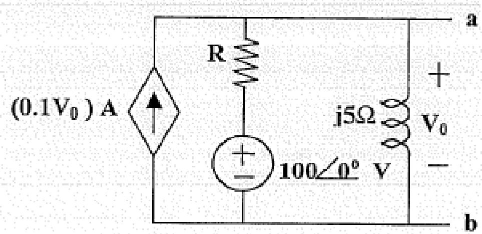
A. 0.2 <29.9°Ω
B. 5Ω
C. 5<-29.9°Ω
D. 1.8<-27.8°Ω
E. None of the above.

3. In the circuit shown, V_{AB} =48.3<30 V. Find V_{in}

a. 50<135 V
b. 36<45 V
c. 80<135 V
d. 73<45 V
e. None of the above



11. Considering the circuit below find its Thevenin impedance between a and b.

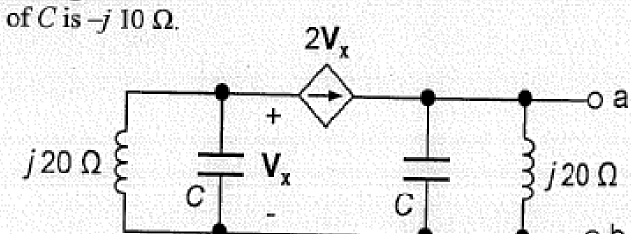


R= 2Ω

a) (2 + j1)Ω
b) (2 + j4)Ω
c) (j5)Ω
d) (j2)Ω
e. None of the above

19. Determine Thevenin's impedance looking into terminals ab, given the reactance of C is $-j$ 10 Ω.

A. $-j$ 20 Ω
B. $+j$ 20 Ω
C. $-j$ 40 Ω
D. $+j$ 40 Ω
E. None of the above



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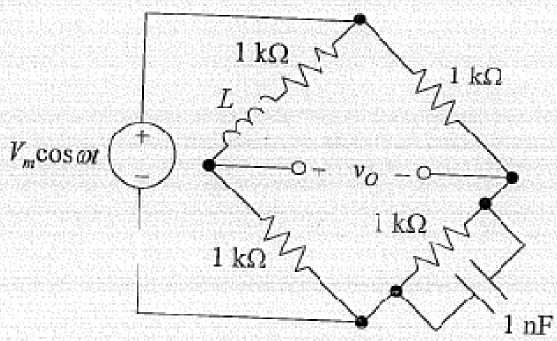
6. Determine L so that the bridge is balanced ($v_O = 0$) at $\omega = 10^6$ rad/s.

A. 1 mH B. 2 μH

C. 4 μH D. 1 H

E. None of the above



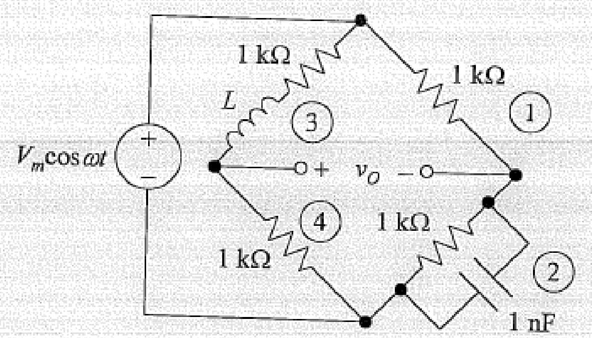
Solution: : At balance, $\dfrac{Z_1}{Z_2} = \dfrac{Z_3}{Z_4}$;

$Z_2 = \dfrac{R / j\omega C}{R + 1/j\omega C} = \dfrac{R}{1 + j\omega CR}$; Ω.

$\dfrac{Z_1}{Z_2} = 1 + j\omega CR$. Hence. $\dfrac{R + j\omega L}{R} =$

$1 + \dfrac{j\omega L}{R} = 1 + j\omega CR$, or

$L = CR^2 = 10^{-9} \times 10^6 \equiv 1$ mH.



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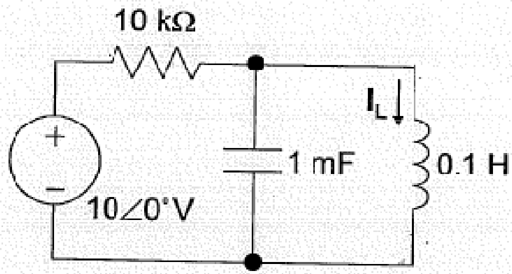
5. Determine $\mathbf{I}_L$, given that $\omega = 100$ rad/s.



A. zero
B. infinite
C. $1\angle 90^\circ$ A
— D. $1\angle -90^\circ$ A
E. None of the above

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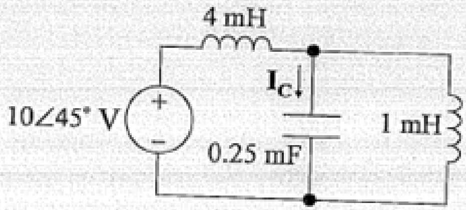
2. Determine $\mathbf{I}_C$, given that $\omega = 2$ krad/s



→ A. $5\angle 135^\circ$ A B. $10\angle 45^\circ$ A

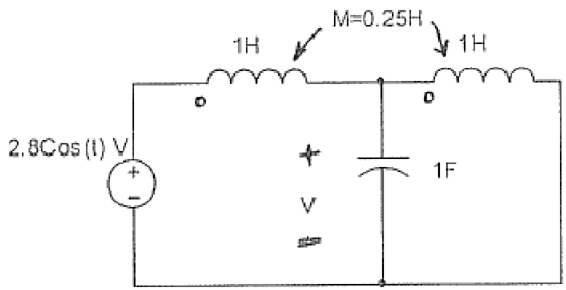
C. $5\angle -45^\circ$ A D. $10\angle 90^\circ$

E. None of the above

Solution: $j\omega L = j2\times 10^{3}\times 10^{-3} = j2\ \Omega$; $\dfrac{1}{j\omega C} = \dfrac{1}{j2\times 10^{3}\times 0.25\times 10^{-3}} = -j2\ \Omega.$

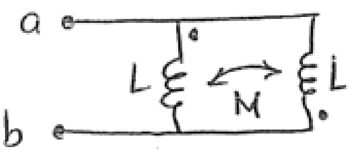
The parallel impedance of $j2\ \Omega$ and $-j2\ \Omega$ is infinite, so that no current flows in the 4 mH inductor. The voltage across the capacitor is $10\angle 45^\circ$ V, and $\mathbf{I}_C = \dfrac{10\angle 45^\circ}{-j2} = 5\angle 135^\circ$ A.

-7- Find the voltage across the capacitor of the circuit shown.



a. Cos(2.26t) b. 0 c. 2.26Cos(t) d. 0.25Cos(t)
f. None of the above

7.Find the equivalent inductance for the following connection , such that: L=60mH, L'=80mH and M=100mH.



a)34.2mH b)86.6mH c)-15.3mH d)134.2mH e)NOA

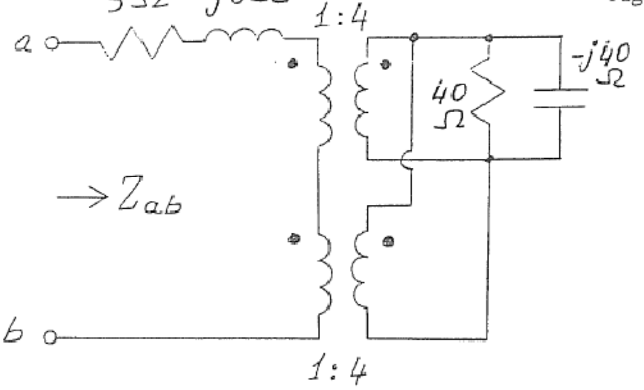


Figure 7.

7. Two identical transformers are connected as shown in figure 7. Determine the impedance Z_{ab}.

A. 10 Ω
B. 15 Ω
C. 10 +j10 Ω
D. 10 – j10 Ω
E. None of the above

3. Calculate the voltages v_1 and v_2 in the circuit of Fig. 2.

A. v_1 = -10 cost V; v_2 = -9 cost V
B. v_1 = 10 cost V; v_2 = 9 cost V
C. v_1 = 10 cost V; v_2 = -9 cost V
D. v_1 = 9 cost V; v_2 = -10 cost V
E. None of the above

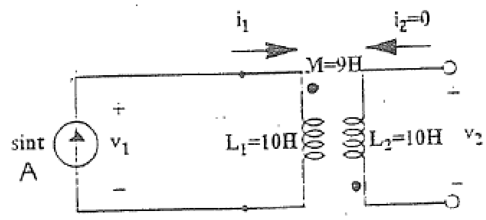


Figure 2.

8. Find the turns ratio for the ideal transformer shown in Fig. 7 required to match the 200 ohms source impedance to the 8 ohms load.

A. n = 3
B. n = 4
C. n = 5
D. n = 6
E. None of the above

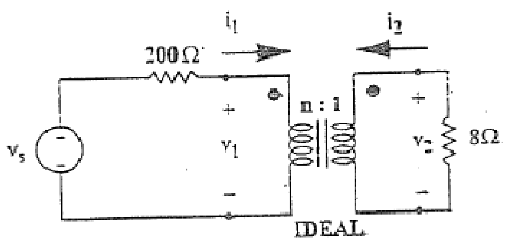


Figure 7.

15. Determine the Thevenin equivalent circuit between terminals a and b in Fig. 13 if $V_s = 10\angle 0^\circ$ V.

A. $V_{Th} = 40$ V; $R_{Th} = 25\ \Omega$
B. $V_{Th} = 20$ V; $R_{Th} = 25\ \Omega$
C. $V_{Th} = 40$ V; $R_{Th} = 50\ \Omega$
D. $V_{Th} = 20$ V; $R_{Th} = 50\ \Omega$
E. None of the above

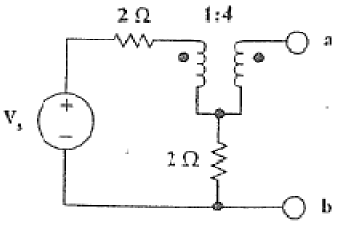


Figure 13

19. Determine L_{eq} in Fig. 16 if $L_3 = 1\ \mu$H.

A. 8 μH
B. 6 μH
C. 4 μH
D. 3 μH
E. None of the above

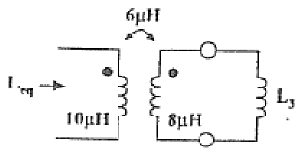


Figure 16

20. If $I_1 = 2$ A in Fig. 17, find the value of I_2 that will minimize the stored energy.

A. 1.33 A
B. 2 A
C. 0
D. 0.67 A
E. None of the above

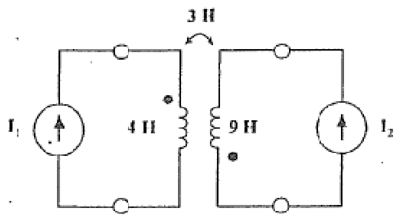
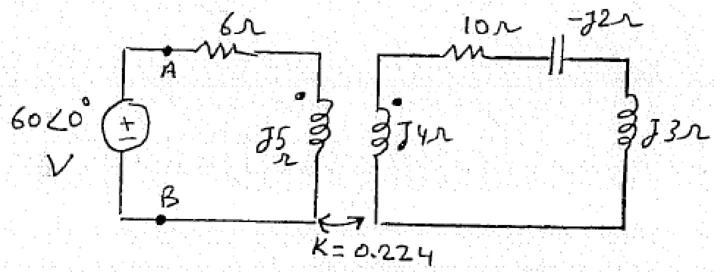


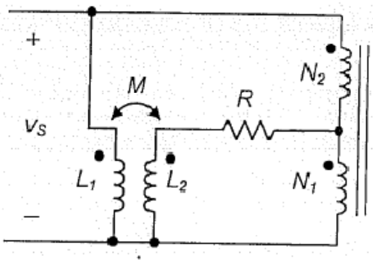
Figure 17

2. Find the input impedance Z_{AB} in the circuit shown below.

A. $6 + j\,5.896\ \Omega$
B. $8.3 + j\,4.7\ \Omega$
C. $6.1 + J\,5\ \Omega$
D. $3.8 + j\,9.2\ \Omega$
E. None of the above

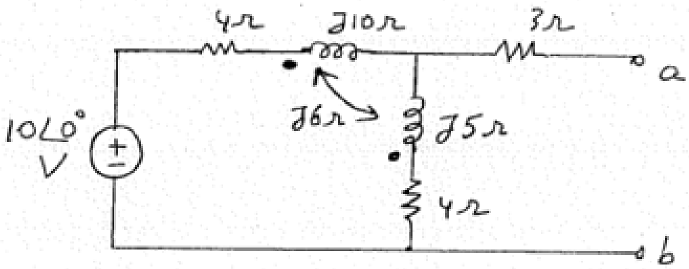


5. In the figure shown, $v_S = 10\cos 100\pi t$ V, $L_1 = 120$ mH, $L_2 = 30$ mH, $R = 100$ ohms, $N_1 = 400$ turns, and $N_2 = 1600$ turns. Determine the coupling coefficient so that no current flows in the 100 ohm resistor.



A. 0.4
B. 0.5
C. 0.6
D. 0.8
E. None of the above

9. In the circuit shown below, find the Thevenin equivalent circuit as seen from terminals a-b.



A. V_{Thev}= 4.82<-34.60, V, Z_{Thev}= 8.62<48.79 Ω
B. V_{Thev}= 4.82< 34.60, V, Z_{Thev}= 8.62<40.38 Ω
C. V_{Thev}= 48.2<-34.60, V, Z_{Thev}= 86.2<48.79 Ω
D. V_{Thev}= 5<-34.60, V, Z_{Thev}= 8.1<48.79 Ω
E. None of the above

12. Consider a source Vs supplying the primary of a transformer. The secondary is connected to a purely capacitive load Zc. The primary impedance is Z1, the secondary impedance is Z2, and the mutual impedance between primary and secondary is Zm. Calculate the currents I1 at primary and I2 at secondary.

Given: Vs = 150 <0° V, Z1=j3600 Ω, Z2=j2500 Ω, Zm=j1200 Ω, Zc= -j2400

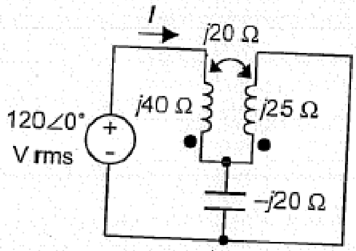
A. I1= 13.9 <-90° mA, I2=166.6<+90° mA
B. I1= 13.9 <0° mA, I2=166.6<+180° mA
C. I1= 33.5 <-90° mA, I2=356.5<+90° mA
D. I1= 33.5 <0° mA, I2=356.5<+180° mA
E. None of the above

Assume dot markings are both up

1. Two magnetically coupled coils have a coefficient of coupling K=0.5. When they are connected in series, their total inductance is 80 mH. When connection of one of the coils is reversed, the total inductance becomes 40 mH. Specify which of the following represents the self-inductance of one of the coils L.

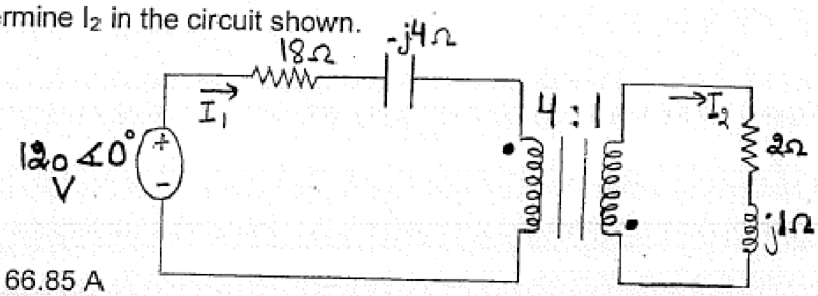
A. 60 mH
B. 52.36 mH
C. 40 mH
D. 5.64 mH
E. None of the above

8. Determine *I*.



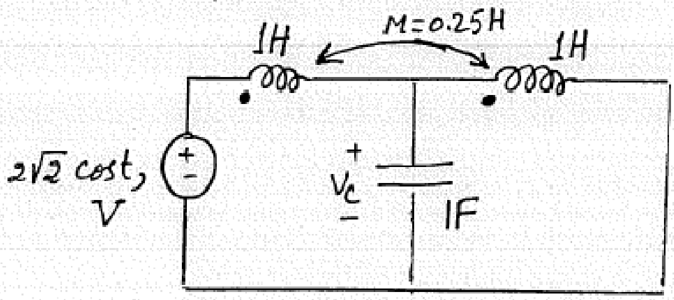
A. +j4 A rms
B. −j6 A rms
C. −j4.8 A rms
D. −j8 A rms
E. None of the above

11. Determine I_2 in the circuit shown.



A. 25.61 <166.85 A
B. 3.56<-166.85 A
C. 16.42<-13.15 A
D. 9.33 <-193.15 A
E. None of the above

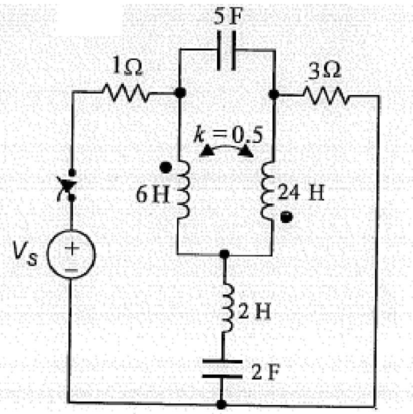
5. Find the voltage $V_c(t)$ across the capacitor of the circuit shown below.



a. 1.6 cost, V
b. 1.41 cost, V
c. 2.26 cos t, V
d. cost, V
e. None of the above

3. Determine the total energy stored in the capacitors and inductors after the switch has been closed for a long time, assuming $V_S = 8$ V.

A. 12 J
B. 30 J
C. 120 J
D. 148 J
E. None of the above



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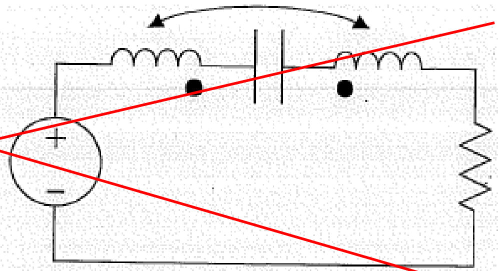
4. If the dot marking on one of the coils is reversed, the damping coefficient α:

A. increases
B. decreases
C. remains the same



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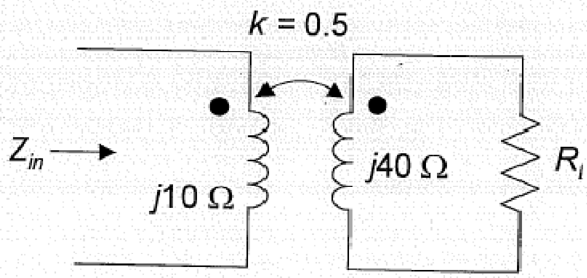
7. Determine the minimum value of Z_{in} as R_L is varied between zero and infinity.

A. $j5\ \Omega$
B. $j7.5\ \Omega$
C. $j10\ \Omega$
D. 0
E. None of the above



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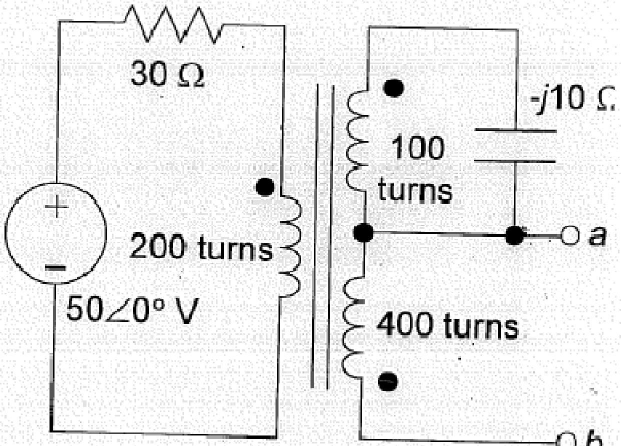
9. Determine Thevenin's equivalent circuit between terminals ab, assuming the transformer is ideal.

$V_{Th} = -64 + j48\ V$

$Z_{Th} = \frac{96}{5}(4 - j3)\ \Omega$



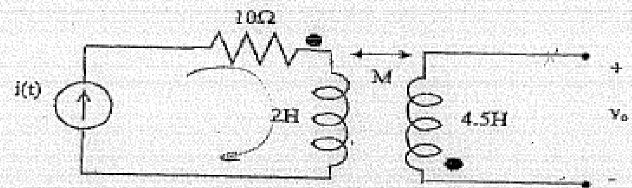
The sinusoidal current source i(t) is given by:

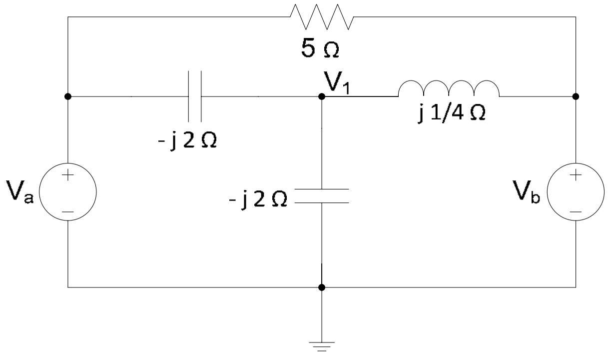
$$i(t) = 10\sin(120\pi t)(\text{Amps})$$
$$t \geq 0$$

This current is applied to the primary coil of a transformer, as shown below. The primary coil (self-inductan
2H) is 100%-coupled to the secondary coil (self-inductance 4.5H).

Find the value of the voltage v_o at $t = 0$.

(a) 15.75 kV
(b) -11.31 kV
(c) -15.75 kV
(d) 11.31kV
(e) None of these





1. Find the correct node-equation for the voltage V_1.

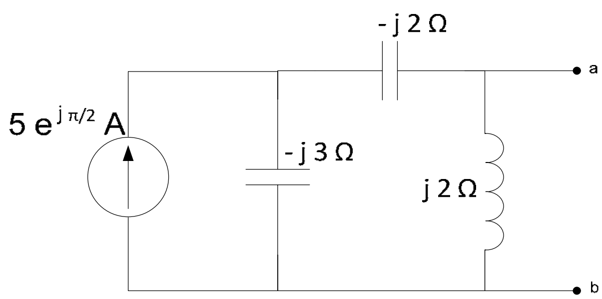
a) $6V_1 + V_a - 8V_b = 0$

b) $2V_1 + V_a - 4V_b = 0$

c) $V_1 - V_a + V_b = 0$

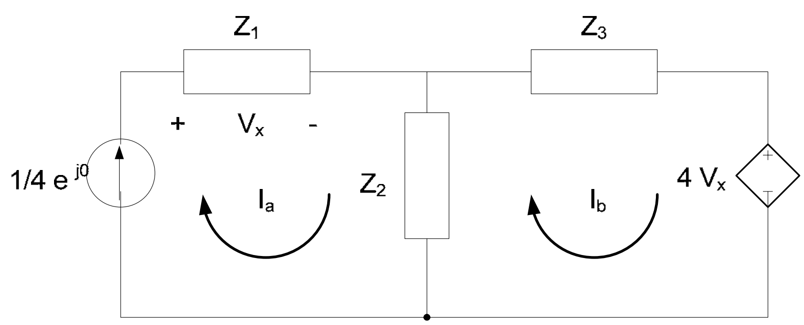
d) $3V_1 - 2V_a + V_b = 0$

e) $7V_1 - 4V_a + V_b = 0$



4. Find the Thevenin equivalent circuit with respect to the terminals a-b. What are the values of V_{Th} in V and Z_{Th} in Ω?

a) V_{Th} =-10 V, Z_{Th} = j10/3 Ω
b) V_{Th} =-8 V, Z_{Th} = j3 Ω
c) V_{Th} =-6 V, Z_{Th} = j14/5 Ω
d) V_{Th} =-4 V, Z_{Th} = j8/3 Ω
e) V_{Th} =-2 V, Z_{Th} = j5/2 Ω



5. What is the expression for V_x?

a) (Z_1+Z_2)
b) $5\,Z_1$
c) $Z_1/4$
d) $2\,Z_1$
e) $Z_1/2$

6. What is the correct set of equations for the mesh currents I_a and I_b?

a) $I_a(-Z_1+4Z_2)-I_b(4Z_2+4Z_3)=0, I_a-5=0$

b) $I_a(-Z_1+2Z_2)-I_b(2Z_2+2Z_3)=0, I_a-2=0$

c) $I_a(-Z_1+Z_2)-I_b(Z_2+Z_3)=0, I_a-1=0$

d) $I_a(-2Z_1+Z_2)-I_b(Z_2+Z_3)=0, I_a-1/2=0$

e) $I_a(-4Z_1+Z_2)-I_b(Z_2+Z_3)=0, I_a-1/4=0$

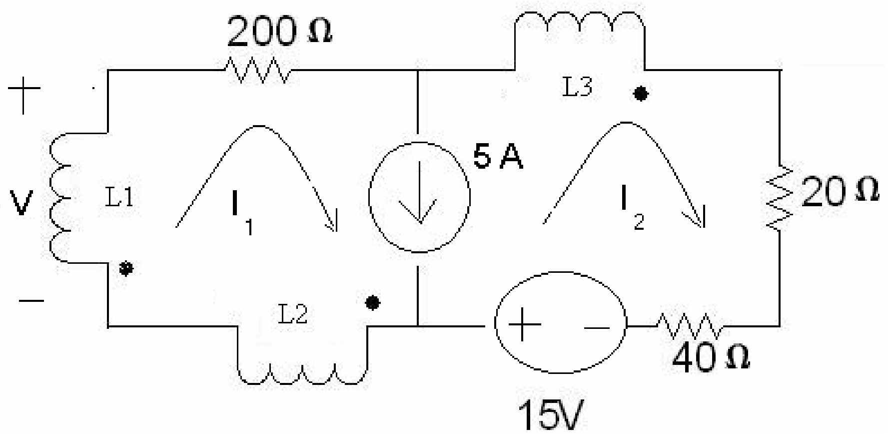
17. If a capacitor with impedance Z_2 is connected in parallel to a load $Z_1 = 300 + j450\ \Omega$. What should be Z_2 in ohms so that the equivalent load is purely resistive?

a) -928.6 j
b) -1112.5 j
c) -650 j
d) -750 j
e) None of the above

22. Assuming that the voltage V across inductance L1 is as shown in figure below and that the mutual inductance between

- L1 and L2 is M12
- L1 and L3 is M13
- L2 and L3 is M23

Use the mesh technique to find the expression of the voltage V.



a) $V = -L_1 dI_1/dt - M_{12} dI_1/dt + M_{13} dI_2/dt$

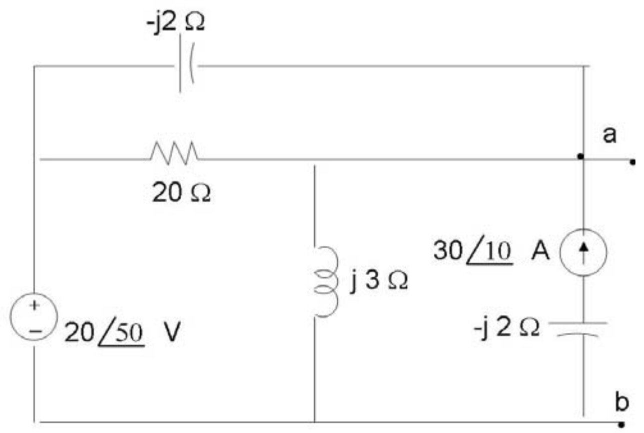
b) $V = -L_1 dI_1/dt + M_{12} dI_1/dt + M_{13} dI_2/dt$

c) $V = -L_1 dI_1/dt + M_{12} dI_1/dt - M_{13} dI_2/dt$

d) $V = -L_1 dI_1/dt + M_{12} dI_1/dt + M_{13} dI_2/dt$

e) None of the above

Problem 1



Find Zth across a and b

A) Z_{th} = 3.85- j 0.77 Ω

→B) Z_{th} = 1.65- j 5.50 Ω

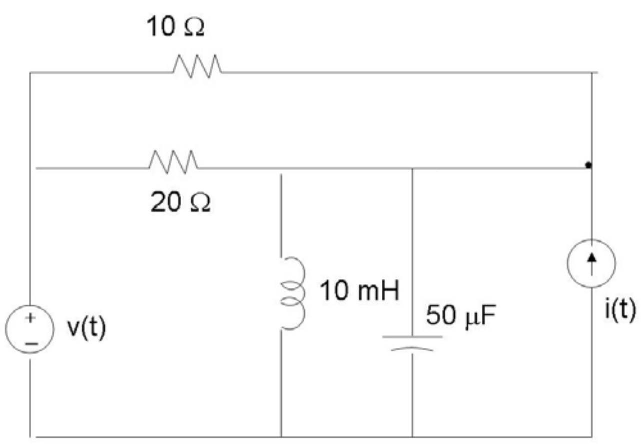
C) Z_{th} = 5.29- j 8.82 Ω

D) Z_{th} = 6.50- j 1.65 Ω

E) None of the above

Problem 2

What are the impedances in this circuit if v(t)=20 cos(10t+50°) Volts and i(t)=50cos(10t+20°) Amperes.



A) 10 Ω, 20 Ω, –j 0.1 Ω, j 0.05 Ω

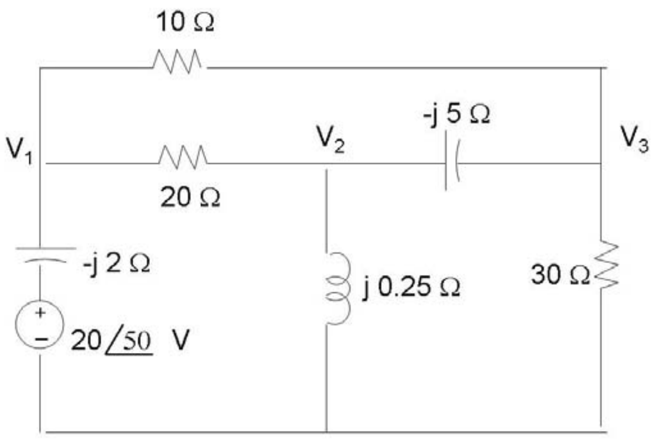
B) 10 Ω, 20 Ω, –j1.0 Ω, j0.05 Ω

→C) 10 Ω, 20 Ω, j 0.1 Ω, –j 2000 Ω

D) 10 Ω, 20 Ω, j 10 Ω, –j 20 Ω

E) None of the above

Problem 3
Find the node equations for the following circuit



A)
$$\begin{aligned}&(0.15+j0.5)V_1-0.05\,V_2-0.1V_3+7.66-j6.43=0\\&-0.05\,V_1+(0.05-j3.8)V_2-j0.2V_3=0\\&-0.1V_1-j0.2\,V_2+(0.133+j0.2)V_3=0\end{aligned}$$

B)
$$\begin{aligned}&(0.15+j0.5)V_1-0.1V_2-0.05\,V_3+7.66-j6.43=0\\&-0.1V_1+(0.1-j3.8)V_2-j0.2V_3=0\\&-0.05\,V_1-j0.2\,V_2+(0.0833+j0.2)V_3=0\end{aligned}$$

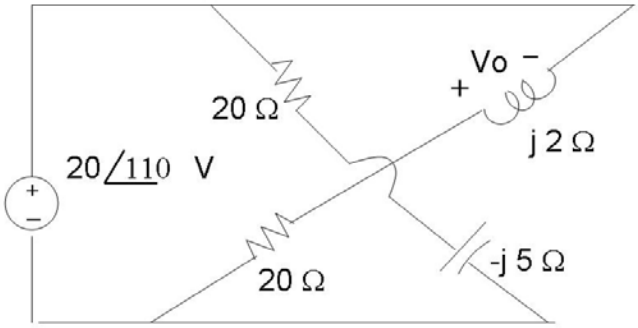
C)
$$\begin{aligned}&(0.15+j0.2)V_1-0.05\,V_2-0.1V_3-12.85-j15.32=0\\&-0.05\,V_1+(0.05-j3.8)V_2-j0.2V_3=0\\&-0.1V_1-j0.2\,V_2+(0.133+j0.2)V_3=0\end{aligned}$$

D)
$$\begin{aligned}&(0.15+j0.2)V_1-0.1V_2-0.05\,V_3-12.85-j15.32=0\\&-0.1V_1+(0.1-j3.8)V_2-j0.2V_3=0\\&-0.05\,V_1-j0.2\,V_2+(0.0833+j0.2)V_3=0\end{aligned}$$

E) None of the above

Problem 4

Find V_0 (t) given ω=120 rad/sec.



A) $V_0 = -0.99 \cos(120t +94.29.^{o})$ Volts

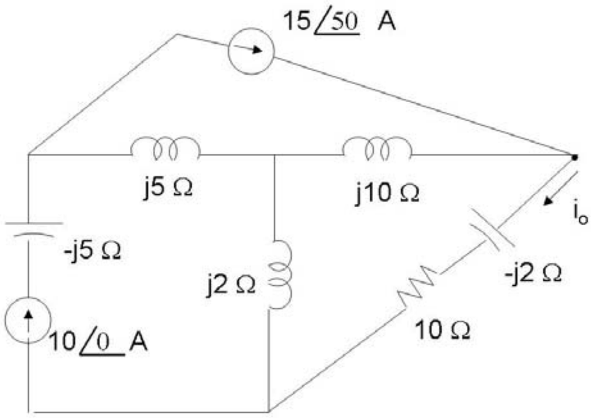
→ B) $V_0 = -1.99 \cos(120t +194.29^{o})$ Volts

C) $V_0 = -1.99 \cos(120t -25.7^{o})$ Volts

D) $V_0 = -0.99 \cos(120t -115.71^{o})$ Volts

E) None of the above

Problem 5



Find i_o in the circuit above.

A) $8.178\angle 104.62^{0}$

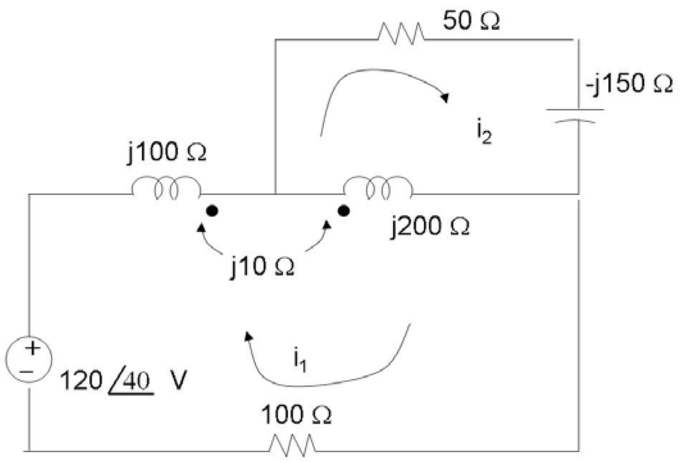
B) $23.14\angle 89.62^{0}$

C) $16.36\angle 104.62^{0}$

→ D) $11.57\angle 89.62^{0}$

E) None of the above

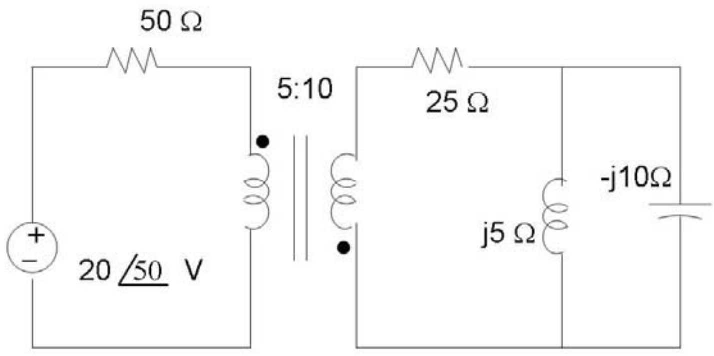
Problem 6



Given the circuit above, what are the two mesh equations?

A) $-120\angle 40^0 + (100 + j280)\, i_1 - 190\, i_2 = 0; \quad - j190\, i_1 + (50 + j50)\, i_2 = 0$
B) $-120\angle 40^0 + (100 + j400)\, i_1 - 250\, i_2 = 0; \quad - j250\, i_1 + (50 + j50)\, i_2 = 0$
C) $-120\angle 40^0 + (100 + j200)\, i_1 - 150\, i_2 = 0; \quad - j150\, i_1 + (50 + j50)\, i_2 = 0$
D) $-120\angle 40^0 + (100 + j320)\, i_1 - 210\, i_2 = 0; \quad - j210\, i_1 + (50 + j50)\, i_2 = 0$
E) None of the above

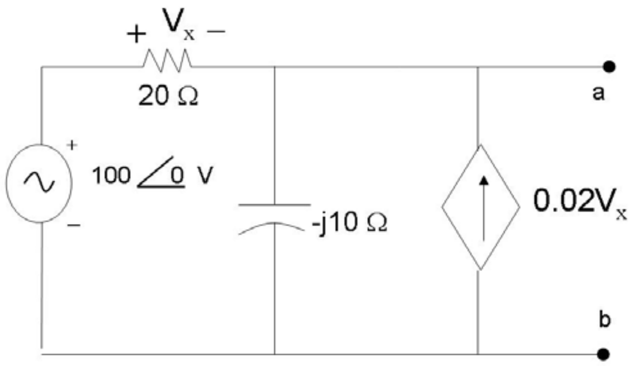
Problem 7



In the circuit shown above, what is the value of the reflected impedance of the 50 ohms resistor from the primary to the secondary side?

A) 100 Ω
B) 12.5 Ω
C) 25 Ω
D) 200 Ω
E) None of the above

Problem 8



In the circuit shown above, find the Thevenin voltage across a,b

A) $76.82\angle -39.80^0 \quad V$

B) $57.3\angle -55.0^0 \quad V$

C) $28.6\angle -63.0^0 \quad V$

D) $65.99\angle -48.3^0 \quad V$

E) None of the above

Problem 9

For the same circuit of previous problem, find Zth across a,b

A) Zth= 4.9- j4.1 Ω

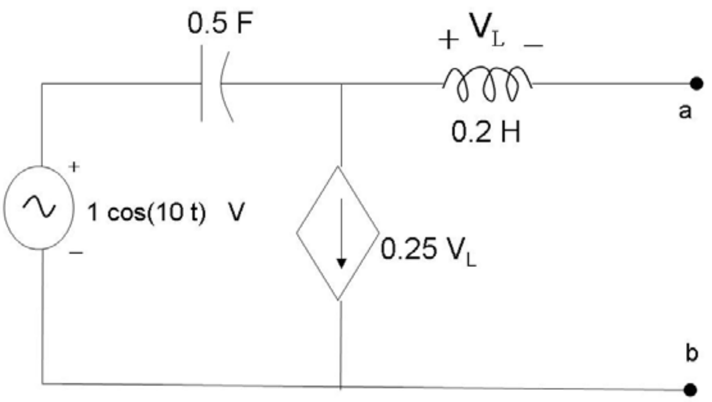
B) Zth= 1.08- j2.12 Ω

C) Zth= 4.7- j6.71 Ω

D) Zth= 3.7- j5.1 Ω

E) None of the above

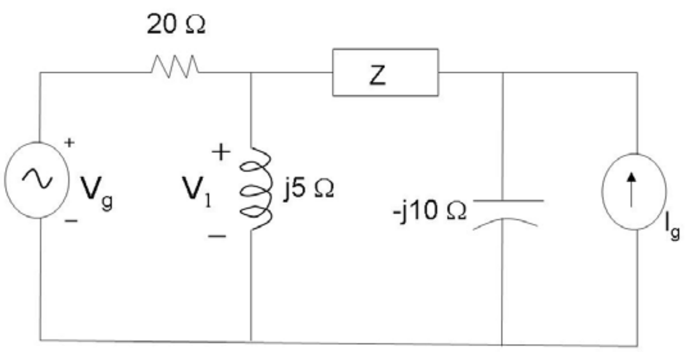
Problem 10



Find the Thevenin equivalent resistance and capacitance/inductance with respect to the terminals a,b in the circuit shown above

→ A) R = 0.1Ω; L=0.18 Ω
B) R = 0.2Ω; L=0.38 Ω
C) R = 0.25Ω; L=0.43 Ω
D) R = 0.15Ω; L=0.36 Ω
E) None of the above

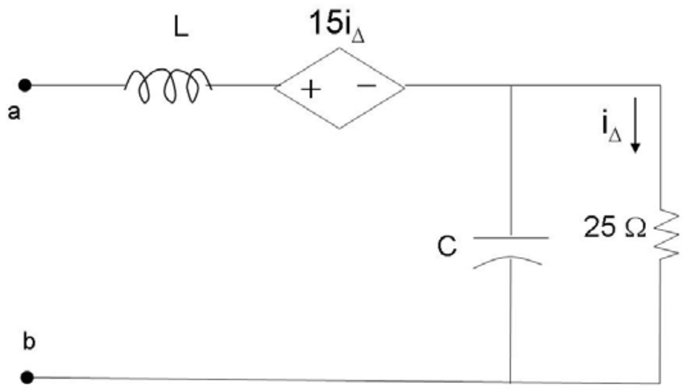
Problem 11



In the circuit shown above, find the value of the impedance Z if
. $V_1 = 40 + j30 \quad V$, $V_g = 100 - j50 \quad V$, and $I_g = 20 + j30 \quad A$

A) 10–j5 Ω
B) 58+j14 Ω
→C) 68+j24 Ω
D) 5+j20 Ω
E) None of the above

Problem 12



Find the input impedance Zi at the terminals a,b in the circuit shown above

A) $Z_i = jL\omega + \dfrac{40}{1+j25C\omega}$ Ω

B) $Z_i = jL\omega + \dfrac{25}{1+j40C\omega}$ Ω

C) $Z_i = jL\omega + \dfrac{15}{1+j40C\omega}$ Ω

D) $Z_i = jL\omega + \dfrac{40}{1+j15C\omega}$ Ω

E) None of the above

Problem 13

In the circuit of the previous problem, find the frequency ω such that the input impedance Zi is purely resistive.

A) $\omega = \dfrac{1}{40C}\sqrt{1000\dfrac{C}{L}-1}$ rad/s

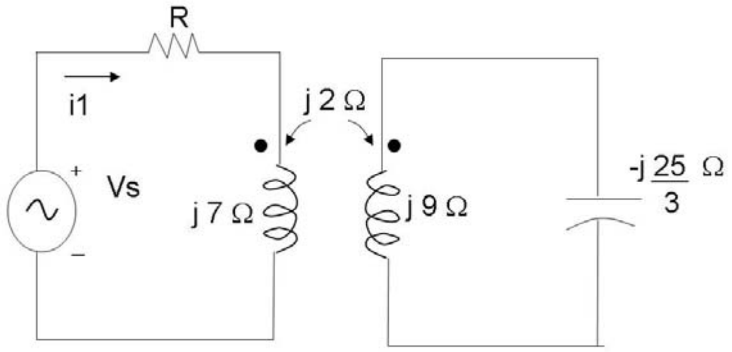
B) $\omega = \dfrac{1}{25C}\sqrt{1000\dfrac{C}{L}-1}$ rad/s

C) $\omega = \dfrac{1}{40C}\sqrt{600\dfrac{C}{L}-1}$ rad/s

D) $\omega = \dfrac{1}{15C}\sqrt{600\dfrac{C}{L}-1}$ rad/s

E) None of the above

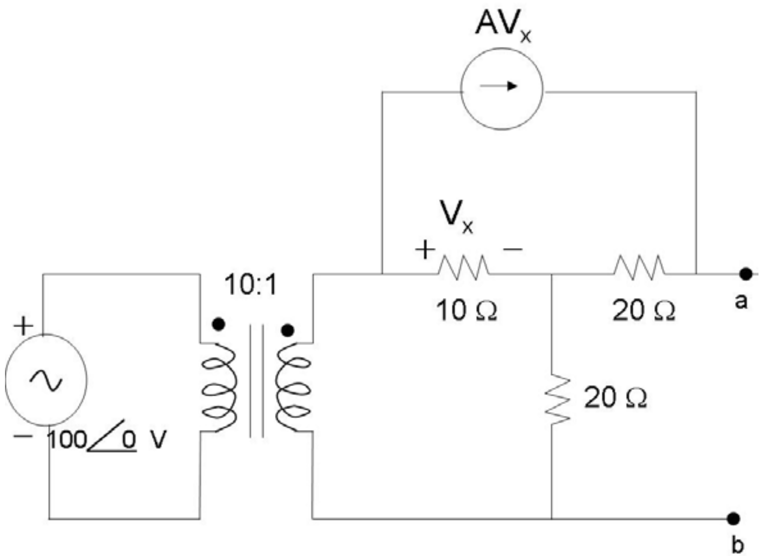
Problem 14



In the circuit shown above, it is given that **R=1 Ω,** and Vs= $10\angle 0$ volts. Find the current i1 as indicated.

A) $8\angle -53.13^0$ A
B) $7.07\angle -53.13^0$ A
C) $7.07\angle -45^0$ A
D) $8\angle -45^0$ A
E) None of the above

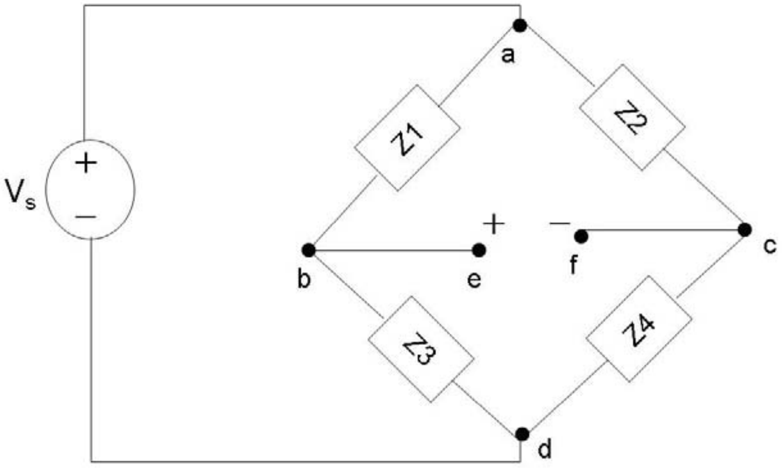
Problem 5



Find the magnitude of the Thevenin Voltage V_{th} across terminals a,b in the circuit above. Given A=1/4.

A) 15.0 V
B) 37.5 V
C) 14.29 V
D) 7.15 V
E) None of the above

Problem 7



In the circuit shown above, given $V_s = 48\angle 90^0$ V, $Z_1 = 3 + j4\,\Omega$, $Z_2 = 8 - j6\,\Omega$, $Z_3 = 3 - j4\,\Omega$ and $Z_4 = 8 + j6\,\Omega$. The Thevenin equivalent circuit values for the voltage souce and the internal impedance across terminals e and f are:

A) $14\angle 0^0$ V, $3.5 - j3.5\,\Omega$

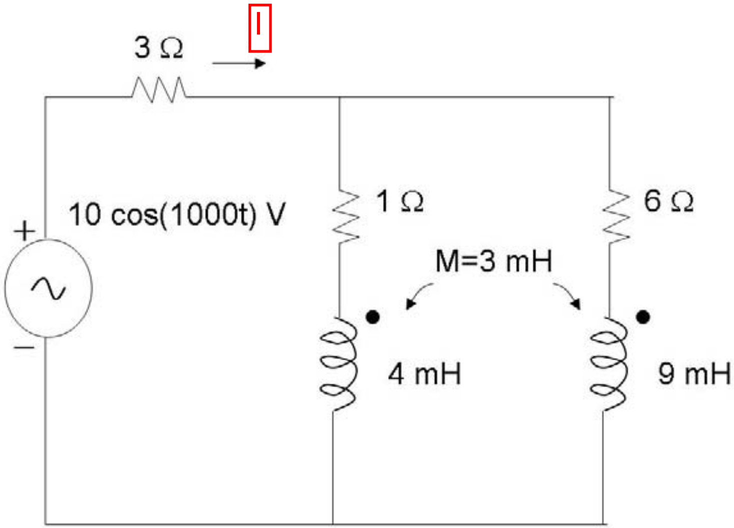
B) $50\angle 0^0$ V, $2.5 + j2.5\,\Omega$,

C) $14\angle 0^0$ V, $7.29\,\Omega$

→D) $50\angle 0^0$ V, $10.42\ \Omega$

E) None of the above

Problem 8



In the circuit shown above, the phasor form of the current I in amperes is:

A) $1.833\angle -45.0^{0}\quad V$

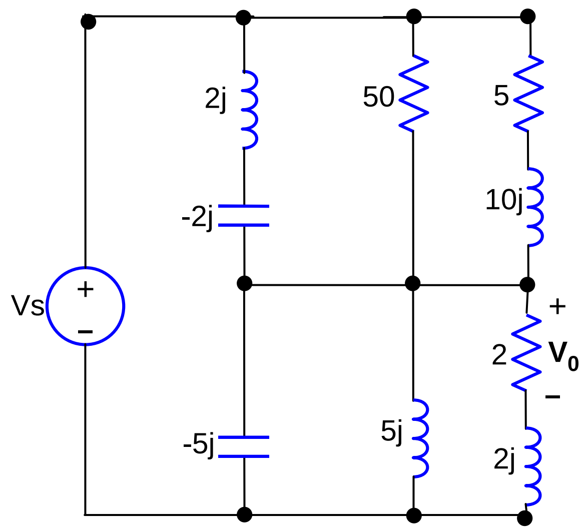
B) $0.917\angle -45.0^{0}\quad V$

C) $1.5\angle -53.13^{0}\quad V$

D) $3.0\angle -53.13^{0}\quad V$

E) None of the above

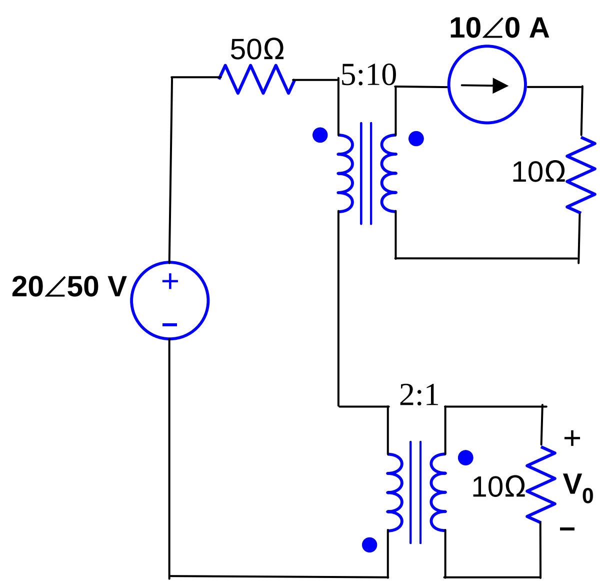
Problem 12



Find V_0 if the source voltage is Vs = $20\angle 60^{o}$ Volts.

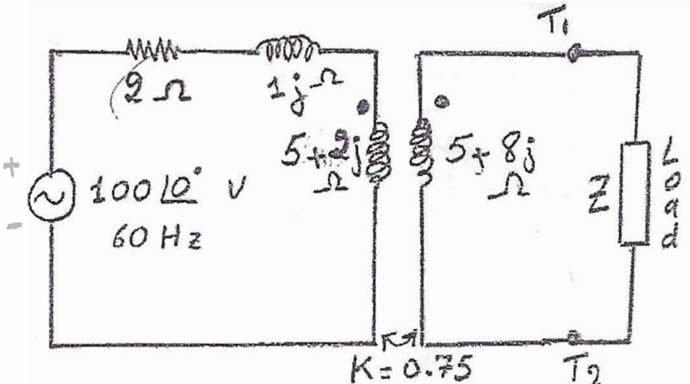
A) $14.14 \angle 15^{o}$ V
B) $7.07\angle 15^{o}$ V
C) $20\angle 60^{o}$ V
D) $10 \angle 60^{o}$ V
E) None of the above

Problem 13



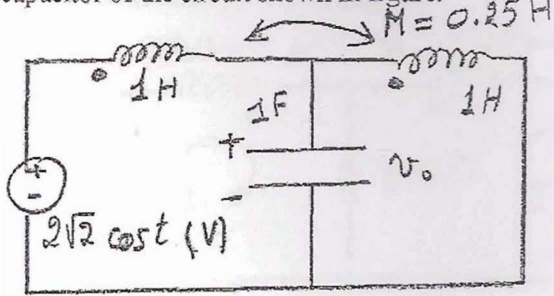
Find V_0.
A) $400 \angle 0$ V
B) $- 400 \angle 0$ V
C) $100 \angle 0$ V
D) $- 100 \angle 0$ V
E) None of the above

3. Determine the Thevenin impedance to the left of the terminals T1-T2 of the circuit shown in figure.



a. $15\angle 50^0\ \Omega$
b. $5+5j\ \Omega$
c. $1.086+2.535j\ \Omega$
d. $9.69\angle 51.07^0\ \Omega$
e. None of the above.

5. Find the voltage $v_0(t)$ across the capacitor of the circuit shown in figure.



a. $1.60\cos(2t)$ V.

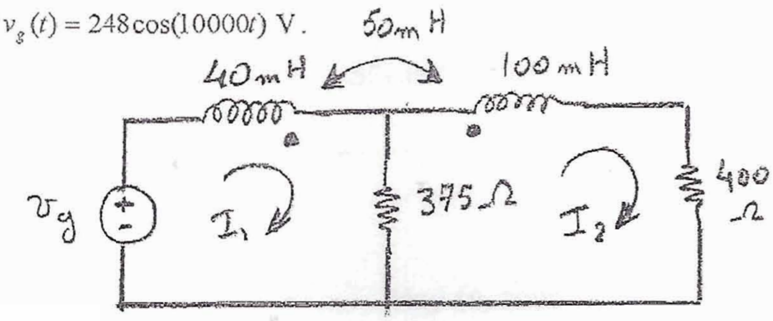
b. $1.60\sin(t)$ V.

c. $3.2\cos(t)$ V.

d. $2.26\cos(t)$ V.

e. None of the above.

9. Find I_1 in the figure shown if $v_g(t) = 248\cos(10000t)$ V.

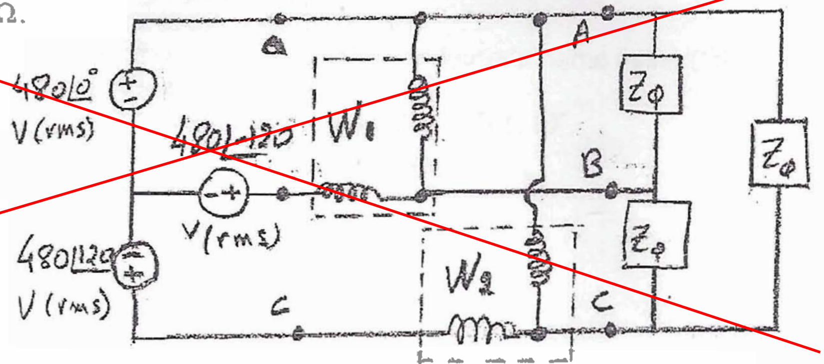


a. $0.8 - 0.62j$ A.
b. $0.4 - 0.3j$ A.
c. $0.6 - 0.6j$ A.
d. $0.9 - 0.31j$ A.
e. None of the above.

10. In problem 9, find the average power delivered to the 375 Ω resistor.

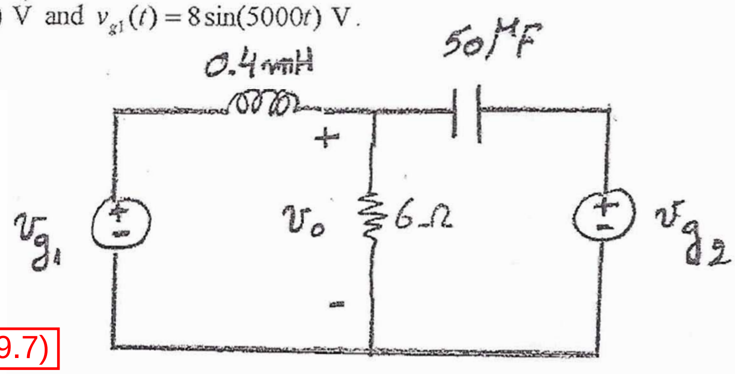
a. 99.2 W.
b. 50.3 W.
c. 49.2 W.
d. 62.7 W.
e. None of the above.

11. Calculate the reading of the two wattmeters ($W_1 + W_2$) in the circuit shown in figure. The value of $Z_\phi = 60\angle 30^0\ \Omega$.



a. ~10 kW.
b. ~29.9 kW.
c. ~33.2 kW.
d. ~15.4 kW.
e. None of the above.

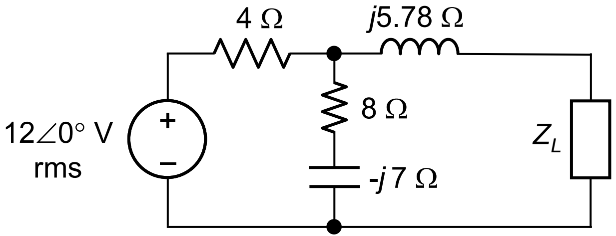
12. Find the steady-state expression for $v_o(t)$ in the circuit shown if $v_{g1}(t) = 10\cos(5000t + 53.13^o)$ V and $v_{g1}(t) = 8\sin(5000t)$ V.



a. $4\cos(5000t)$ V.
b. $8\cos(5000t)$ V.
c. $16\cos(5000t)$ V.
d. $12\cos(5000t)$ V.
e. None of the above.

22.3cos(5,000t + 29.7)

Problem 8 (14 pts)
Consider the circuit shown



a. For $Z_L = 3 - j5.2\ \Omega$,determine the average power developed by the voltage source and the average power absorbed by the load. (4 pts)

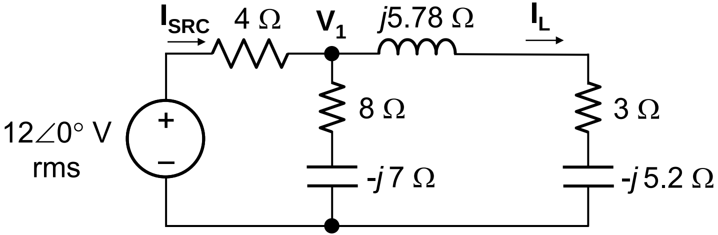
$$\mathbf{V_{Th}} = 12\frac{8 - j7}{12 - j7} = 9 - j1.74\text{ V};$$

$|\mathbf{V_{Th}}| = 9.18$ V;

$\mathbf{I_L} = \mathbf{V_{Th}}/6 = 1.5 - j0.29$ A;

$\mathbf{V_1} = (3 - j\,0.58)I_L = 4.68 + j\,0$ V

$$\mathbf{I_{SRC}} = \frac{12 - 4.68}{4} = 1.83 + j\,0\text{ A}$$



$$P_{SRC} = V_1 I_{SRC} = 12 \times 1.83 \cong 22\text{ W};\ P_L = \frac{(9.18)^2}{4 \times 3} \cong 7\text{ W}.$$

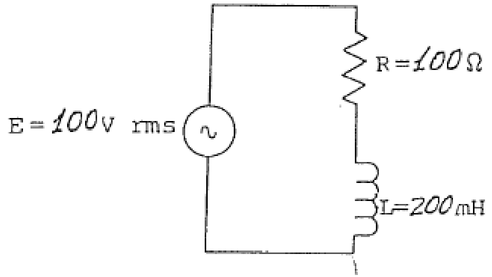


Figure 3.

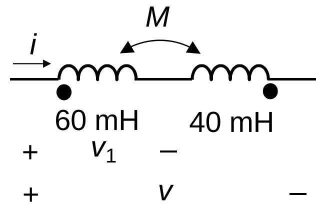
3. Determine the power dissipated in the load in the circuit shown in figure 3. f = 60 Hz.

A. 38.8 W
B. 63.8 W
C. 52.5 W
D. 45.3 W
E. None of the above

1. A coil (R and L) has a resistance of 10Ω and draws a current of 5A (RMS) when connected across a 100V (RMS), 60 Hz source. Determine the inductance of the coil.

a. 17.32 mH
b. 32.48 mH
c. 45.94 mH
d. 102.73 mH
e. None of the above

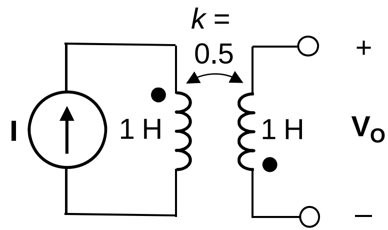
1. If M = 5 mH, determine the ratio v_1/v.



Solution: $L_{eq} = L_1 + L_2 - 2M$; $v = L_{eq}\dfrac{di}{dt}$; $v_1 = L_1\dfrac{di}{dt} - M\dfrac{di}{dt}$;

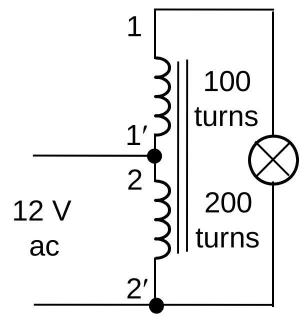
hence, $\dfrac{v_1}{v} = \dfrac{L_1 - M}{L_1 + L_2 - 2M} = \dfrac{60 - M}{100 - 2M}$

2. Determine $\mathbf{V_o}$, given that $\mathbf{I} = 1\angle 0°$ A and $\omega = 10$ rad/s.



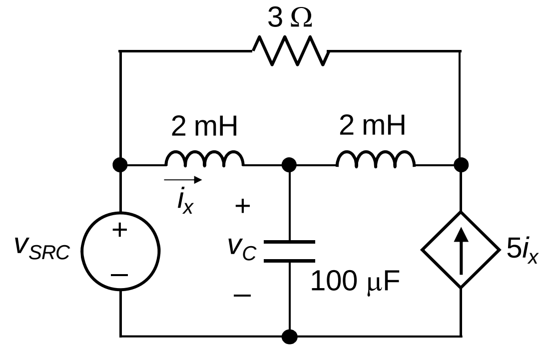
Solution: $M = k\sqrt{L_1L_2} = 0.5\,\text{H}$; secondary voltage is $j\omega M\mathbf{I}$, with the dotted terminal positive with respect to the undotted terminal. Hence, $\mathbf{V_o} = -j\omega M\mathbf{I} = -j10\times0.5\mathbf{I} = -j5\mathbf{I}$.

3. The lamp glows brighter when the dots are at coil terminals



Solution: The lamp glows brighter when the voltage across it is largest. This occurs when the voltages across the windings are additive, that is, when the dots are at terminals 1 and 2 or 1′ and 2′.

6. Derive the time-domain expression for v_C, given that v_{SRC} = 10sin(2,000t) V.

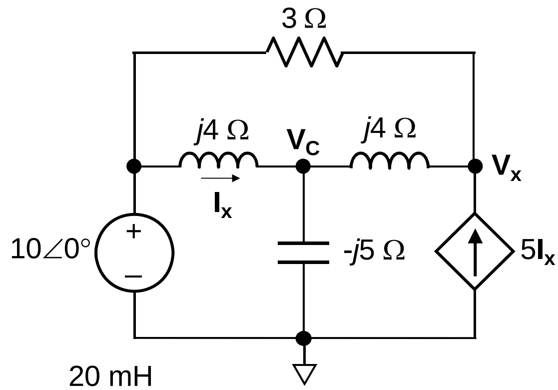


Solution: $\omega L = 2\times10^3\times2\times10^{-3} = 4\ \Omega$;

$\dfrac{1}{\omega C} = \dfrac{1}{2\times10^3\times100\times10^{-6}} = 5\ \Omega$; $\mathbf{V_{SRC}} = 10\angle0°$.

The node-voltage method can be applied, the circuit being as shown. At the middle node:

$\mathbf{V_C}/\text{-}j5 + (\mathbf{V_C} - \mathbf{V_x})/j4 + (\mathbf{V_C} - 10)/j4 = 0$



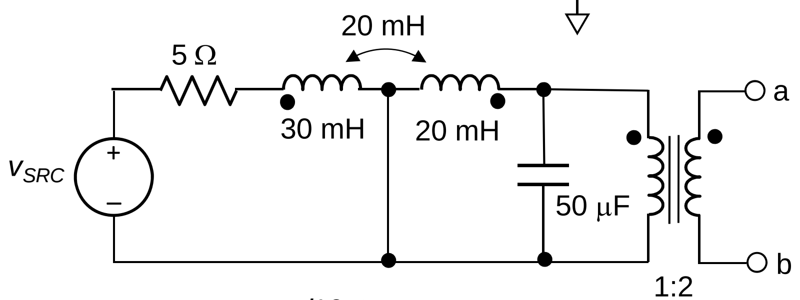
At the right-hand node:

$(\mathbf{V_x} - \mathbf{V_C})/j4 + (\mathbf{V_x} - 10)/3 = 5\mathbf{I_x} = 5(10 - \mathbf{V_C})/j4$

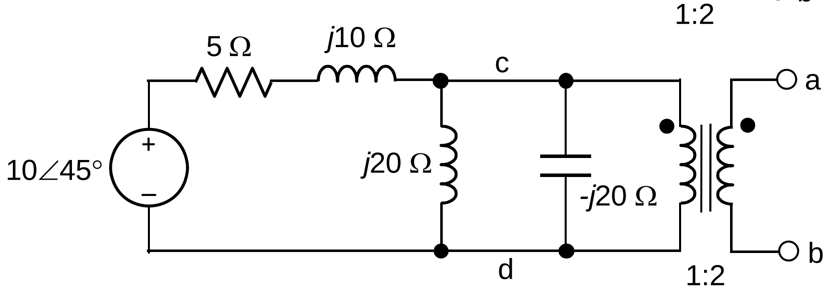
Solving, $\mathbf{V_C} = 11.98 + j1.44 = 12.1\angle6.86°$, so that

$v_C = 12.1\sin(2{,}000t + 6.86°)$ V.

7. Derive $\mathbf{V_{Th}}$ and Z_{Th} as seen between terminals ab, given that v_{SRC} = 10cos(1,000t + 45°) V.



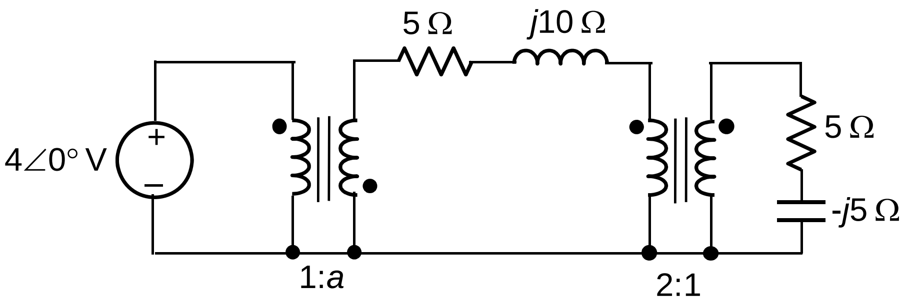
Solution: $\omega L_1 = 10^3\times30\times10^{-3} = 30\ \Omega$; $\omega L_2 = \omega M = 10^3\times20\times10^{-3} = 20\ \Omega$; $\dfrac{1}{\omega C} = \dfrac{1}{10^3\times50\times10^{-6}} = 20\ \Omega$; $\mathbf{V_{SRC}} = 10\angle45°$.



The circuit in the frequency domain will be as shown, where $\omega(L_1 - M) = 10\ \Omega$; $\omega(L_2 - M) = 0\ \Omega$ and is omitted. The $j20\ \Omega$ in parallel with $-j20\ \Omega$ is effectively an open circuit. The current in the $(5 + j10)\ \Omega$ impedance is zero, $\mathbf{V_{cd}} = 10\angle45°$, and $\mathbf{V_{ab}} = \mathbf{V_{Th}} = 20\angle45°$.

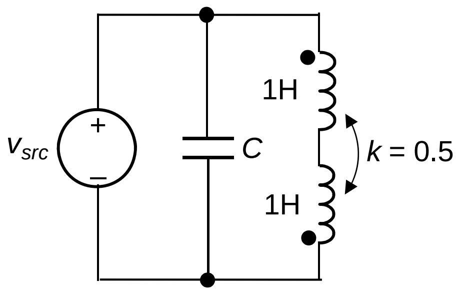
If the independent voltage source is replaced by a short circuit, the impedance on the primary side is (5 + j10) Ω and Z_{Th} = 4(5 + j10) = 20 + j40 Ω.

3. Determine the impedance seen by the source, assuming a = 2.



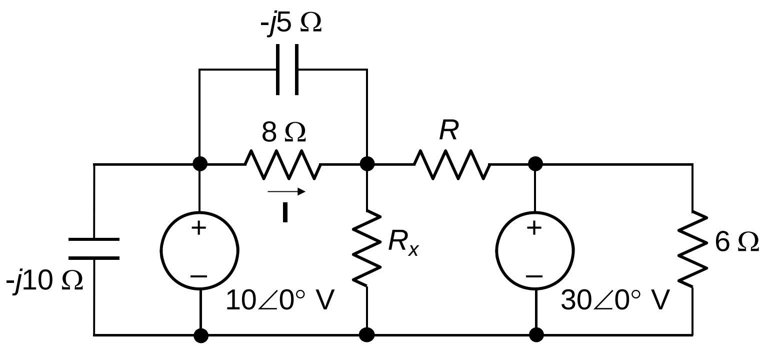
Solution: Reflection of the (5 – j5) Ω through the RH transformer gives (20 – j20) Ω. The impedance on the secondary side of the LH transformer is (25 – j10) Ω. Reflected to the primary side, this becomes (25 – j10)/a^2 Ω.

4. If v_{src} = 10cos(1,000t) V, determine the energy stored in the circuit in the sinusoidal steady state at t = 0, assuming C = 1 μF.



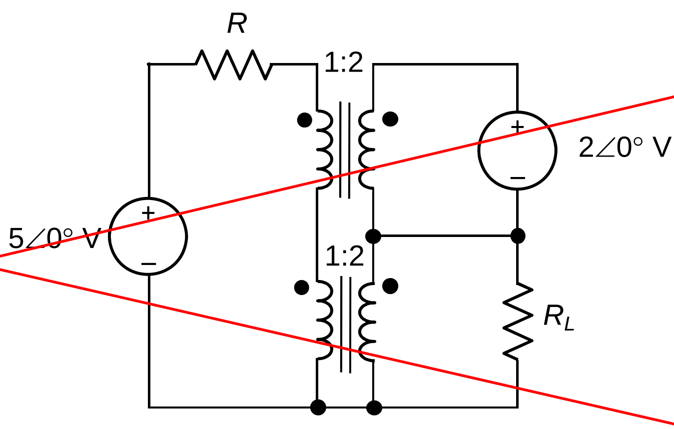
Solution: At t = 0, the voltage across C is 10 V and the current through the inductors is zero, being proportional to the integral of v_{src}. The energy stored is $W = \frac{1}{2}Cv^2 = 50C$.

5. Determine R_x given that **I** = 0 and R = 2 Ω.



Solution: Since **I** = 0, the voltage across R_x is 10 V, and the same current $\frac{30\angle 0^\circ - 10\angle 0^\circ}{R}$ flows through R and R_x. It follows that $\frac{20}{R}R_x = 10$, or $R_x = \frac{R}{2}$.

7. ~~Determine the maximum power that can be delivered to R_L, assuming R = 0.5 Ω.~~



~~**Solution:** The primary voltage of the upper transformer is always 1 V. On~~

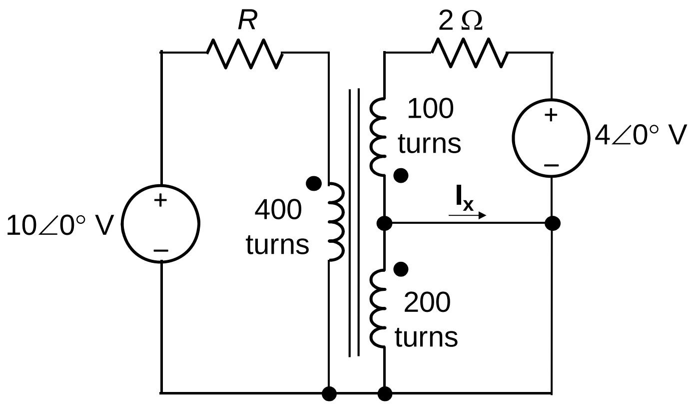
9. Two identical coils, each having an inductance of 10 mH, are connected in series. When the connections to one of the coils are reversed, the total inductance is multiplied by a factor *a*. Determine the coupling coefficient of the coils.

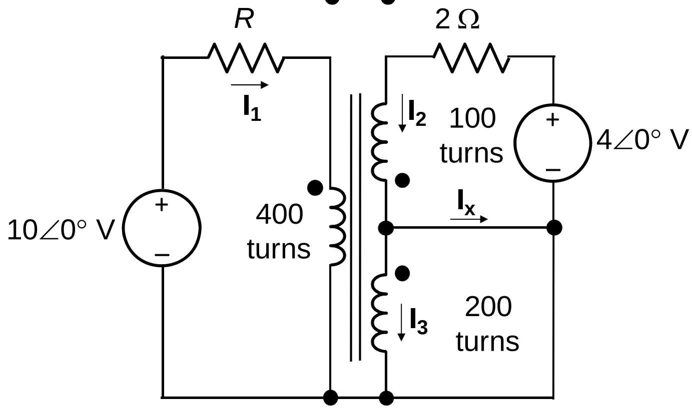
Solution: $(10 + 10 + 2M) = a(10 + 10 - 2M)$; $2M(a + 1) = 20(a - 1)$;

$$M = \frac{10(a-1)}{a+1};\ k = \frac{M}{10} = \frac{(a-1)}{a+1}$$

10. Determine $\mathbf{I_x}$, assuming $R = 4\ \Omega$.

Solution: The voltage across all windings is zero. Hence, $\mathbf{I_1} = \frac{10}{R}$ A, and $\mathbf{I_2} = \frac{4}{2} = 2$ A. Setting the net mmf to zero, $400\mathbf{I_1} - 100\mathbf{I_2} + 200\mathbf{I_3} = 0$, or $\frac{4\times 10}{R} - 2 + 2\mathbf{I_3} = 0$, which gives $\mathbf{I_3} = 1 - \frac{20}{R}$; $\mathbf{I_X} = \mathbf{I_2} - \mathbf{I_3} = 1 + \frac{20}{R}$.

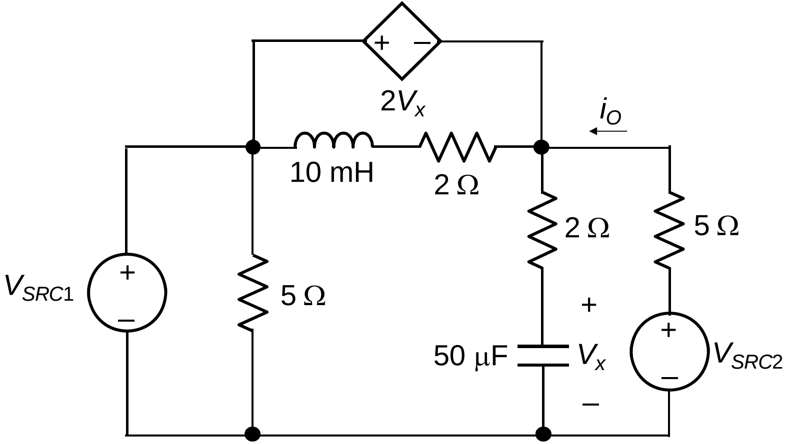




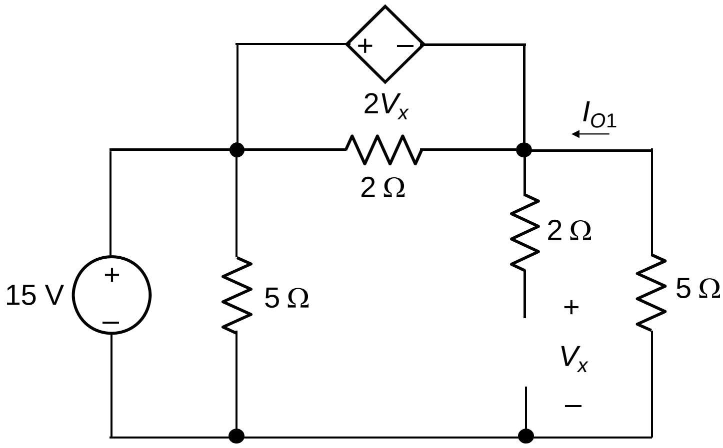
16. Determine i_O, given that

V_{SRC1} is 15 V dc and V_{SRC2} =

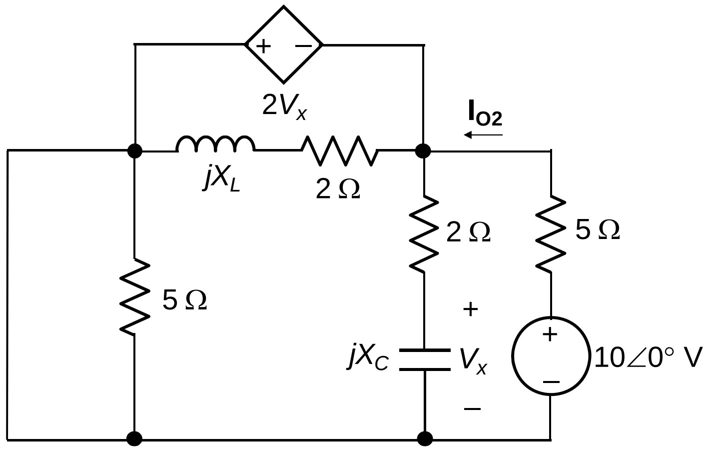
$10\cos(3{,}000t)$ V.



Solution: With V_{SRC1} applied and V_{SRC2} set to zero, the circuit becomes as shown. 15 = $3V_x$, so that V_x = 5 V and $I_{O1} = \frac{-V_x}{5} = -1$ A.



With V_{SRC2} applied and V_{SRC1} set to zero, the circuit becomes as shown. It follows that: $-2V_x = V_x + \frac{2V_x}{jX_C}$, or $V_x\left(3 + \frac{2}{jX_C}\right) = 0$, which gives V_x = 0. Hence, $\mathbf{I_{O2}} = \frac{10\angle 0^\circ}{5} = 2\angle 0^\circ$ A. Thus, i_O = -1 + 2cos(3,000t) A.



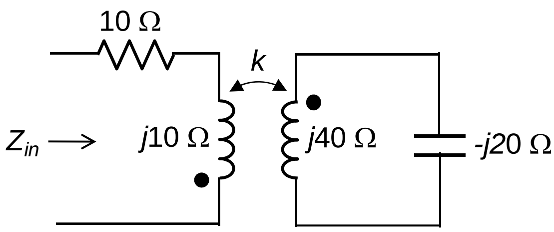
17. Determine k so that the input resistance is purely resistive.

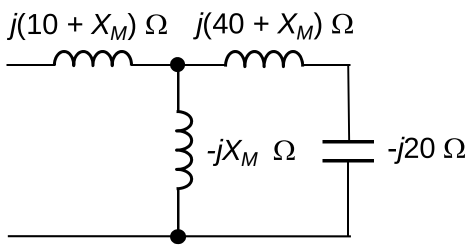
Solution: Disregarding the 10 Ω resistance and replacing the linear transformer by its T-equivalent circuit, the circuit becomes as shown. The input reactance is

$$j10 + jX_M - \frac{jX_M(j20 + jX_M)}{j20} = 0\text{, or}$$

$$10 + X_M - X_M - \frac{X_M^2}{20} = 0\text{, which gives}$$

$$X_M = \sqrt{200} = 10\sqrt{2}\text{. Hence, } k = \frac{10\sqrt{2}}{\sqrt{400}} = \frac{1}{\sqrt{2}} = 0.71.$$





Problem 1

Given 3 elements $R = 10K\Omega$, $L = 10mH$ and $C = 625nF$ powered by a source $v = 90sin(10,000t + \frac{\pi}{4})$ (V). Find the impedance of each element Z_R, Z_L and Z_C.

A) $Z_R = 10K\Omega,\ Z_L = 100j\Omega,\ Z_C = -160j\Omega$

B) $Z_R = 10K\Omega,\ Z_L = 10j\Omega,\ Z_C = -16j\Omega$

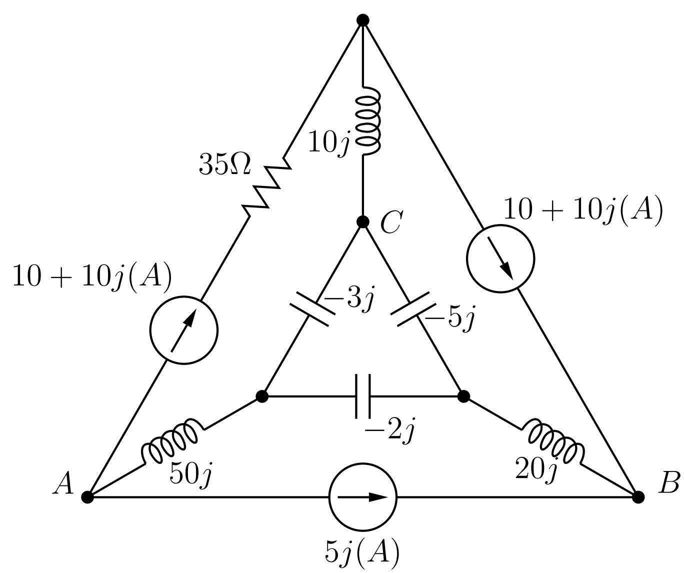
C) $Z_R = 10jK\Omega,\ Z_L = 10j\Omega,\ Z_C = -1600j\Omega$

D) $Z_R = 10K\Omega,\ Z_L = 10j\Omega,\ Z_C = -160j\Omega$

E) None of the above

Problem 2

Find the Thevenin equivalent voltage between A and C. (Impedances are in Ω)



A) 285-190j V

B) -741+494j V

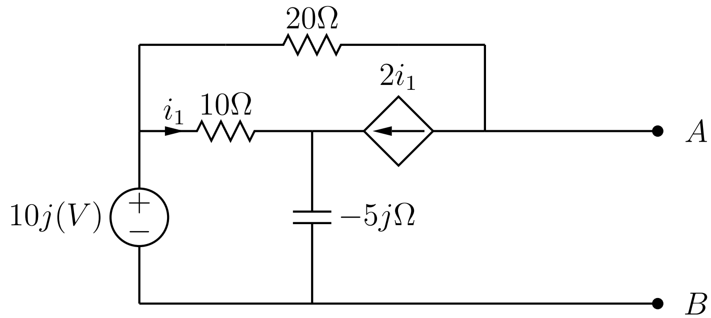
C) -741-494j V

D) 285+190j V

E) None of the above

Problem 3

Find the Thevenin equivalent voltage between A and B.



A) $39.7V\angle 21.6^o$

B) $18.6V\angle 7.1^o$

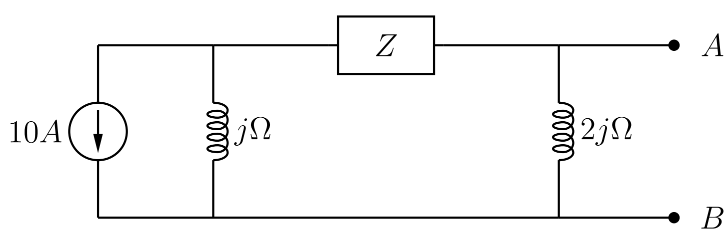
→ C) $18.6V\angle -7.1^o$

D) $39.7V\angle -21.6^o$

E) None of the above

Problem 4

Find the nature of Z such that the Thevenin equivalent impedance between A and B is 1Ω.



→ A) $0.8 - 1.4j\Omega$

B) $0.8 + 1.4j\Omega$

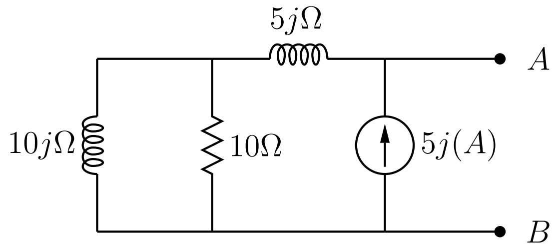
C) $0.5 - 2.5j\Omega$

D) $0.5 + 2.5j\Omega$

E) None of the above

Problem 5

Find the Thevenin voltage between A and B.



A) 50+25j V

B) -100+50j V

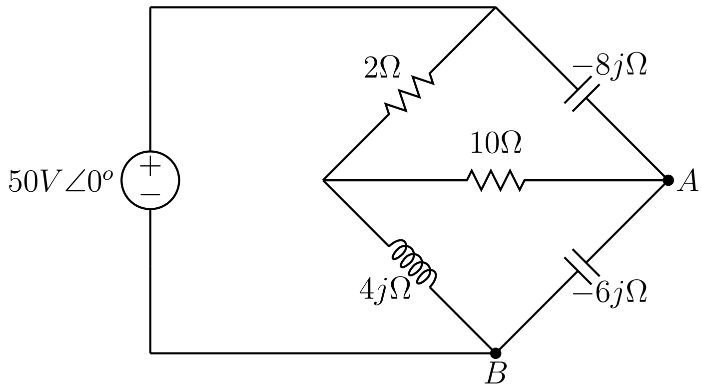
C) 100+50j V

D) -50+25j V

E) None of the above

Problem 6

Find the Thevenin impedance between A and B.



A) 1.49-0.55j Ω

B) 0.96+3.21j Ω

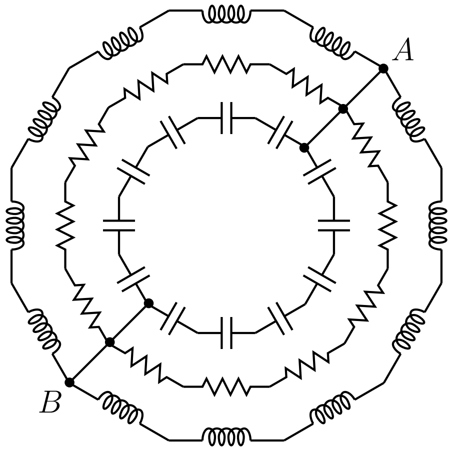
C) 0.96-3.21j Ω

D) 1.49+0.55j Ω

E) None of the above

Problem 7

All the inductors are equal to $5j\Omega$, all the capacitors are equal to $-6j\Omega$, all the resistances are equal to 10Ω. Find Z_{AB}.



A) 30.56+16.98j Ω

B) 30.56-16.98j Ω

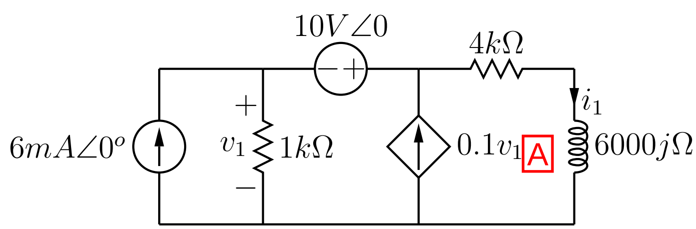
C) 27-9j Ω

D) 27+9j Ω

E) None of the above

Problem 8

Find i_1.



A) 0.76-1.14j mA

B) -0.26+1.6j mA

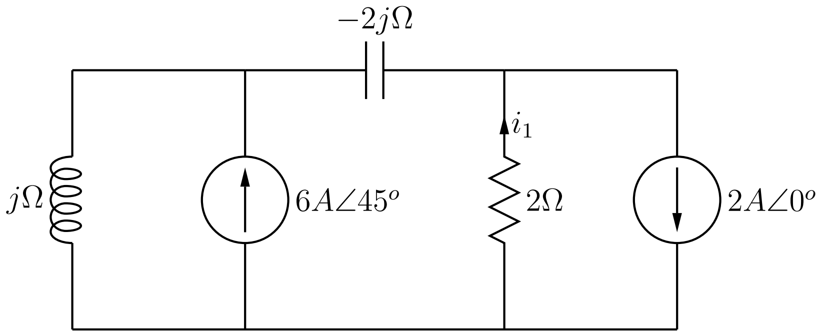
C) 0.26-1.6j mA

D) -0.76+1.14j mA

E) None of the above

Problem 9

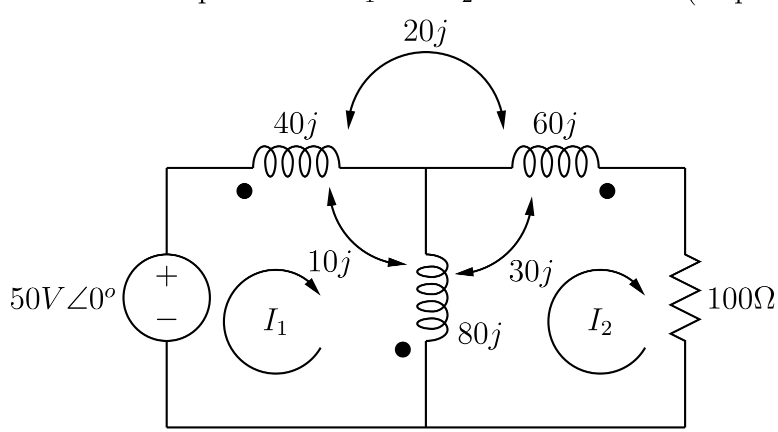
Find i_1.



A) $3.38\angle-29.2^o$ (A)
B) $7.55\angle-82.4^o$ (A)
C) $7.55\angle82.4^o$ (A)
D) $3.38\angle29.2^o$ (A)
E) None of the above

Problem 10

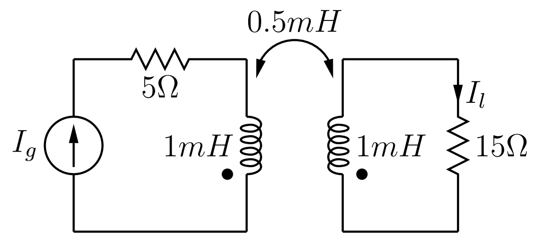
Write the two mesh current equation for I_1 and I_2 **Don't solve**. (Impedances are in Ω).



A) $100jI_1 + 60jI_2 = 50 \qquad 60jI_1 + (100+80j)I_2 = 0$
B) $120jI_1 - 80jI_2 = 50 \qquad -80jI_1 + (100+80j)I_2 = 0$
C) $100jI_1 - 80jI_2 = 50 \qquad -80jI_1 + (100+80j)I_2 = 0$
D) $100jI_1 - 60jI_2 = 50 \qquad -60jI_1 + (100+80j)I_2 = 0$
E) None of the above

Problem 11

If $I_g = 20cos(10,000t + \frac{\pi}{3})(A)$ find the energy associated with the 2 coils at the time $t = 100\pi\mu s$.



A) 65.3mJ

B) 261.3mJ

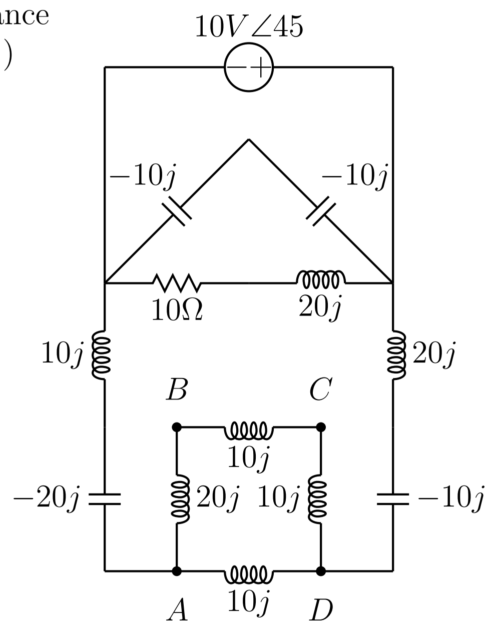
C) 40.7mJ

D) 163mJ

E) None of the above

Problem 12

Find the Thevenin equivalent impedance between A and B. (Impedances are in Ω.)



A) 10j Ω

B) 8j Ω

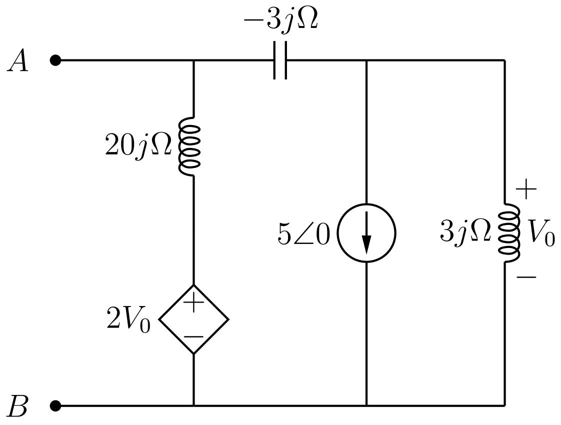
C) 7.5j Ω

D) 12j Ω

E) None of the above

Problem 1

Find the Thevenin equivalent voltage between A and B (V_{AB}).



A) 15j V

B) -20j V

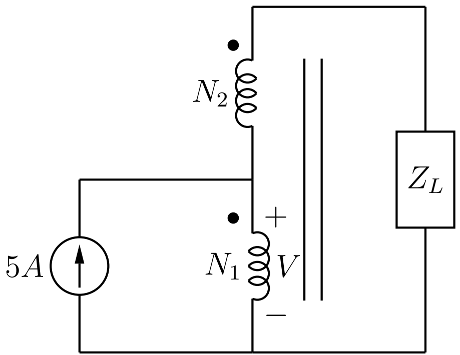
C) 20j V

D) -15j V

E) None of the above

Problem 9

Given $Z_L = 100 + 100j$, N_2=90, N_1=10, find V.



A) $7.07V\angle - 135^o$

B) $63.64V\angle 45^o$

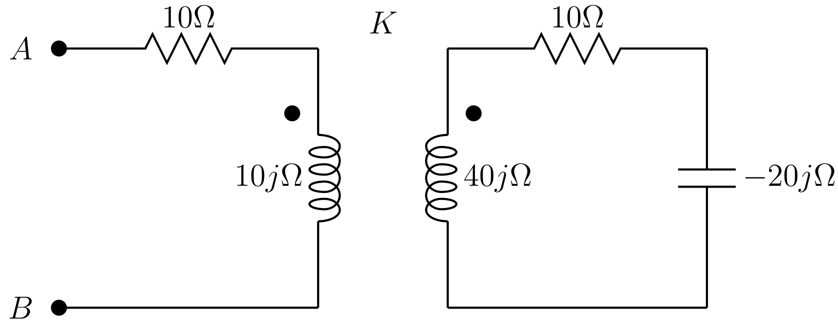
C) $63.64V\angle - 135^o$

D) $7.07V\angle 45^o$

E) None of the above

Problem 16

Consider the linear transformer of the figure below, given $\omega = 1 rad/s$, find the coupling coefficient K, such that the Thevenin impedance between A and B is purely resistive.



→ A) 0.79

B) 0.82

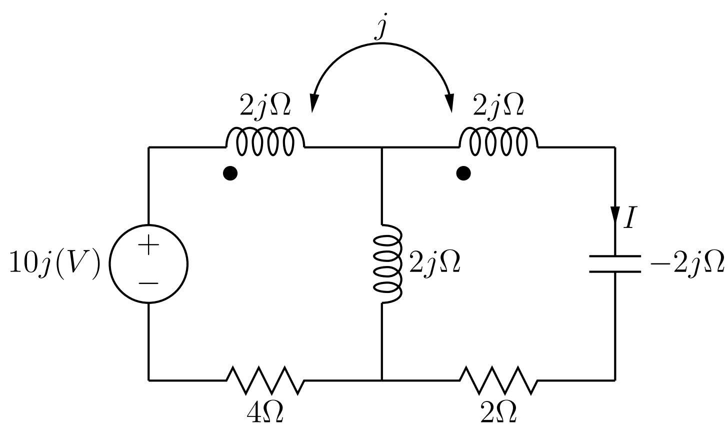
C) 0.85

D) 0.88

E) None of the above

Problem 17

Find I.



A) 0.0389 - 0.6226j A

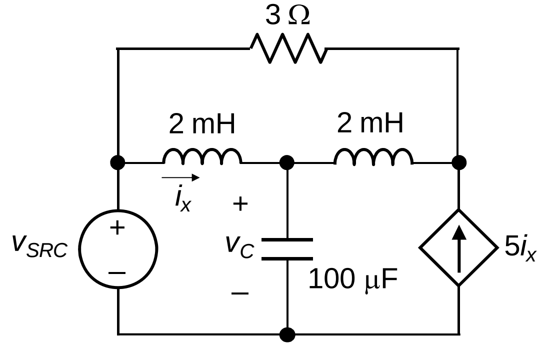
B) -0.0778 + 1.2452j A

→ C) -0.0389 + 0.6226j A

D) 0.0778 - 1.2452j A

E) None of the above

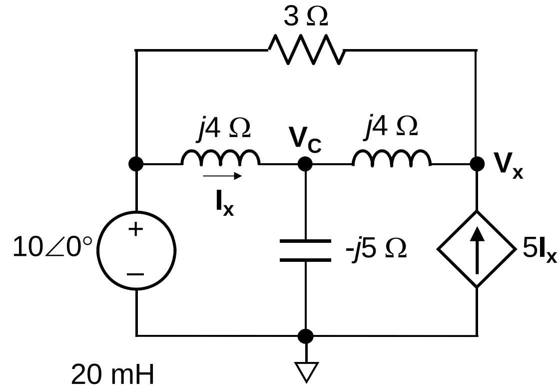
6. Derive the time-domain expression for v_C, given that $v_{SRC} = 10\sin(2{,}000t)$ V.



Solution: $\omega L = 2\times10^3\times2\times10^{-3} = 4\ \Omega$;

$\dfrac{1}{\omega C} = \dfrac{1}{2\times10^3\times100\times10^{-6}} = 5\ \Omega$; $\mathbf{V_{SRC}} = 10\angle0°$.

The node-voltage method can be applied, the circuit being as shown. At the middle node:



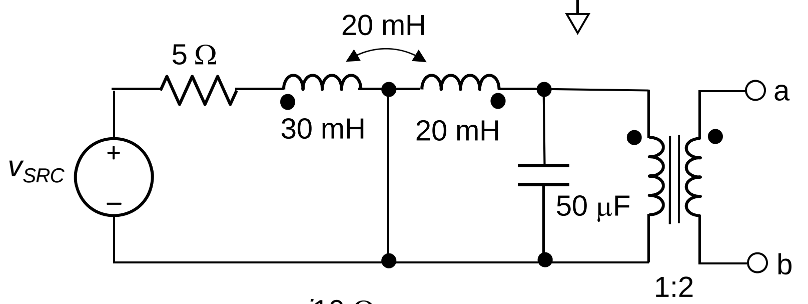
$\mathbf{V_C}/\text{-}j5 + (\mathbf{V_C} - \mathbf{V_x})/j4 + (\mathbf{V_C} - 10)/j4 = 0$

At the right-hand node:

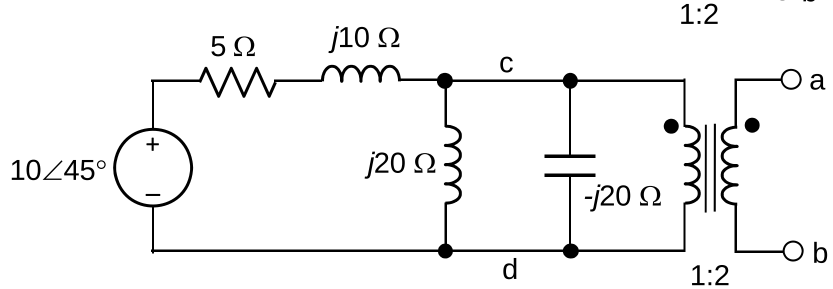
$(\mathbf{V_x} - \mathbf{V_C})/j4 + (\mathbf{V_x} - 10)/3 = 5\mathbf{I_x} = 5(10 - \mathbf{V_C})/j4$

Solving, $\mathbf{V_C} = 11.98 + j1.44 = 12.1\angle6.86°$, so that $v_C = 12.1\sin(2{,}000t + 6.86°)$ V.

7. Derive $\mathbf{V_{Th}}$ and Z_{Th} as seen between terminals ab, given that $v_{SRC} = 10\cos(1{,}000t + 45°)$ V.



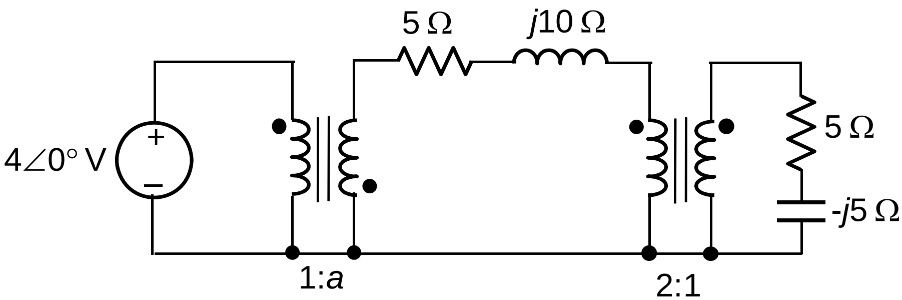
Solution: $\omega L_1 = 10^3\times30\times10^{-3} = 30\ \Omega$; $\omega L_2 = \omega M = 10^3\times20\times10^{-3} = 20\ \Omega$; $\dfrac{1}{\omega C} = \dfrac{1}{10^3\times50\times10^{-6}} = 20\ \Omega$; $\mathbf{V_{SRC}} = 10\angle45°$.



The circuit in the frequency domain will be as shown, where $\omega(L_1 - M) = 10\ \Omega$; $\omega(L_2 - M) = 0\ \Omega$ and is omitted. The $j20\ \Omega$ in parallel with $-j20\ \Omega$ is effectively an open circuit. The current in the $(5 + j10)\ \Omega$ impedance is zero, $\mathbf{V_{cd}} = 10\angle45°$, and $\mathbf{V_{ab}} = \mathbf{V_{Th}} = 20\angle45°$.

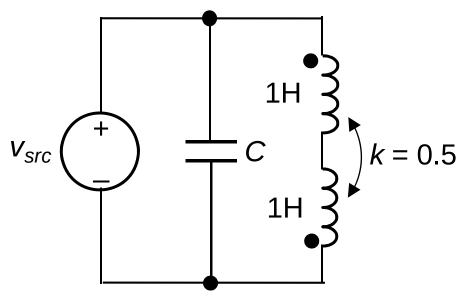
If the independent voltage source is replaced by a short circuit, the impedance on the primary side is $(5 + j10)$ Ω and $Z_{Th} = 4(5 + j10) = 20 + j40$ Ω.

3. Determine the impedance seen by the source, assuming $a = 2$.



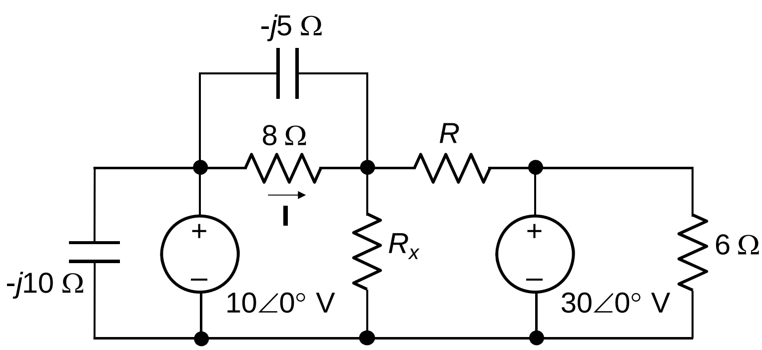
Solution: Reflection of the $(5 – j5)$ Ω through the RH transformer gives $(20 – j20)$ Ω. The impedance on the secondary side of the LH transformer is $(25 – j10)$ Ω. Reflected to the primary side, this becomes $(25 – j10)/a^2$ Ω.

4. If $v_{src} = 10\cos(1{,}000t)$ V, determine the energy stored in the circuit in the sinusoidal steady state at $t = 0$, assuming $C = 1$ μF.

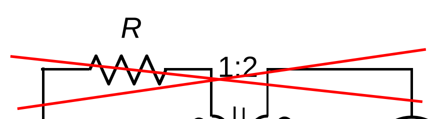


Solution: At $t = 0$, the voltage across C is 10 V and the current through the inductors is zero, being proportional to the integral of v_{src}. The energy stored is $W = \frac{1}{2}Cv^2 = 50C$.

5. Determine R_x given that $\mathbf{I} = 0$ and $R = 2$ Ω.



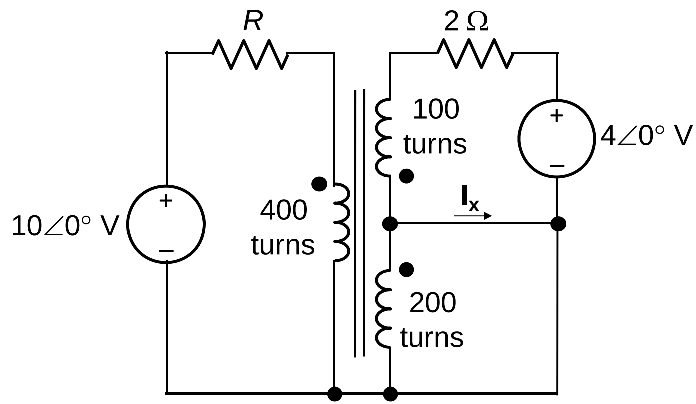
Solution: Since $\mathbf{I} = 0$, the voltage across R_x is 10 V, and the same current $\dfrac{30\angle 0^\circ - 10\angle 0^\circ}{R}$ flows through R and R_x. It follows that $\dfrac{20}{R}R_x = 10$, or $R_x = \dfrac{R}{2}$.



9. Two identical coils, each having an inductance of 10 mH, are connected in series. When the connections to one of the coils are reversed, the total inductance is multiplied by a factor a. Determine the coupling coefficient of the coils.

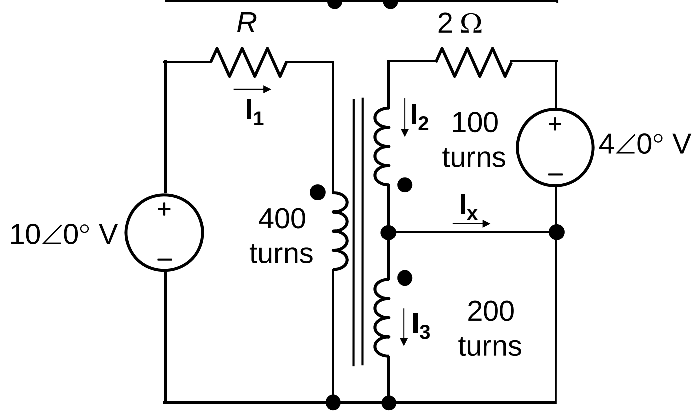
Solution: $(10 + 10 + 2M) = a(10 + 10 - 2M)$; $2M(a + 1) = 20(a - 1)$;

$M = \frac{10(a-1)}{a+1}$; $k = \frac{M}{10} = \frac{(a-1)}{a+1}$



10. Determine $\mathbf{I_x}$, assuming $R = 4\ \Omega$.

Solution: The voltage across all windings is zero. Hence, $\mathbf{I_1} = \frac{10}{R}$ A, and $\mathbf{I_2} = \frac{4}{2} = 2$ A. Setting the net mmf to zero, $400\mathbf{I_1} - 100\mathbf{I_2} + 200\mathbf{I_3} = 0$, or $\frac{4\times 10}{R} - 2 + 2\mathbf{I_3} = 0$, which gives $\mathbf{I_3} = 1 - \frac{20}{R}$; $\mathbf{I_X} = \mathbf{I_2} - \mathbf{I_3} = 1 + \frac{20}{R}$.



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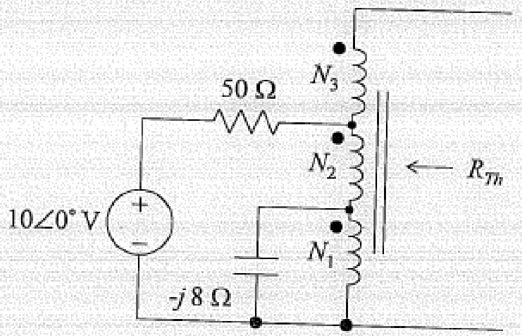
9. Given an ideal autotransformer having three windings of $N_1 = 200$ turns, $N_2 = 300$ turns, and $N_3 = 500$ turns. Determine R_{Th}.

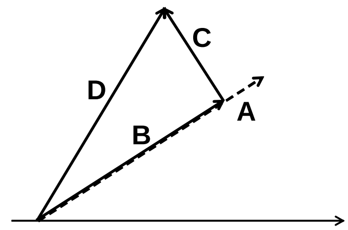
$\boxed{\text{100 ohms}}$



1. A current $i = I_m\cos(\omega t + 30°)$ flows through an impedance $(5 - j5)$ Ω. Determine the rms phasor voltage across the impedance if $I_m = 2.5$ A.

Solution: The phasor current is $I_m\angle 30°$ A. The impedance is $5\sqrt{2}\angle -45^\circ$ Ω. The phasor voltage is $5I_m\sqrt{2}\angle -15^\circ$ V peak value or $5I_m\angle -15^\circ = 12.5\angle -15^\circ$ V rms.

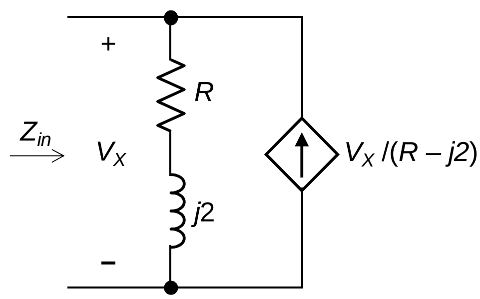
2. In the phasor diagram shown, phasor **A** is a current, the other phasors **B**, **C**, and **D** are voltages. To which of the following combinations of circuit elements does this phasor diagram apply?



A. *R* in series with *L*
B. *R* in series with *C*
C. *R* in parallel with *C*
D. *L* in series with *C*
E. *L* in parallel with *C.*

Solution: The phasor diagram represents a series *RL* circuit, Phasor **B** is the voltage across *R*. Phasor **C** is the voltage across *L*, leading the current by 90°, and phasor **D** is the voltage across the series combination.

3. Determine the input impedance Z_{in} assuming $R = 2$ Ω.

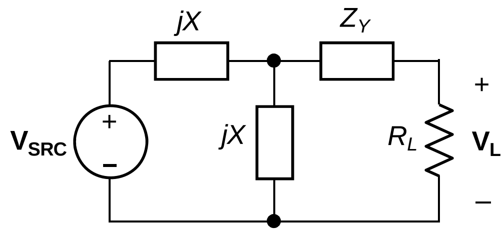


Solution: If a test voltage source $\mathbf{V_T}$ is applied, $\mathbf{I_T}$ $= \left(\frac{1}{R+j2} - \frac{1}{R-j2}\right)\mathbf{V_T}$, or Z_{in} =

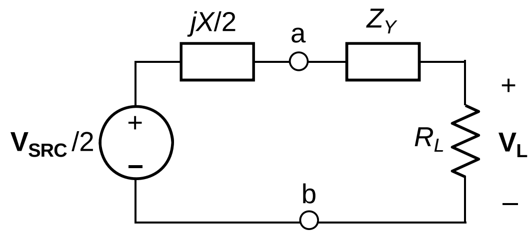
$$\frac{1}{\left(\frac{1}{R+j2} - \frac{1}{R-j2}\right)} = \frac{(R+j2)(R-j2)}{R-j2-R-j2} = \frac{R^2+4}{-j4} = j\left(1+\frac{R^2}{4}\right)$$ Ω. Alternatively, the dependent source is equivalent to an impedance $-(R - j2)$ and $Z_{in} = (R+j2)\,||\,[-(R-j2)] =$

$$-\frac{R^2+4}{j2} = j\left(\frac{R^2}{4}+1\right) = j2\ \Omega.$$

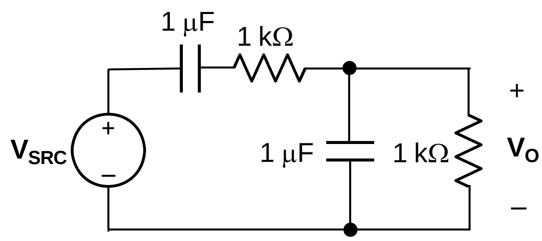
4. Determine Z_Y so that $\mathbf{V_L}$ is in phase with $\mathbf{V_{SRC}}$, assuming X = -5 Ω with R_L and $\mathbf{V_{SRC}}$ unknown.



Solution: TEC as seen from terminals a and b will have Z_{Th} = $jX/2$. For $\mathbf{V_L}$ to be in phase with $\mathbf{V_{SRC}}$, Z_Y = $-jX/2$ = $j2.5$ Ω



5. Determine $\mathbf{V_O}$ if ω = 1 krad/s and $\mathbf{V_{SRC}}$ = 3 V.



Solution: $\omega CR = 1$;

$$\frac{\mathbf{V_O}}{\mathbf{V_{SRC}}} = \frac{R/(1+j\omega CR)}{R/(1+j\omega CR)+R-j/\omega C} = \frac{R/(1+j)}{R/(1+j)+R-j/\omega C} = \frac{1/(1+j)}{1/(1+j)+1-j} =$$

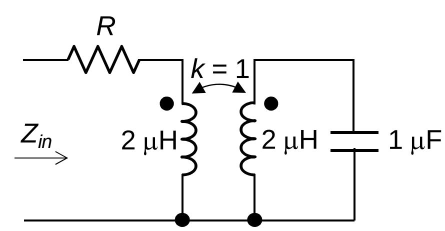
$$\frac{1}{1+(1+j)(1-j)} = \frac{1}{1+1-j^2} = \frac{1}{3}$$; $\mathbf{V_O}$ = $\mathbf{V_{SRC}}/3$ = 1 V.

6. Two coils are wound on a core of high permeability. Coil 1 has 100 turns and coil 2 has 400 turns. A current of 1 A in coil 1, with coil 2 open circuited, produces a core flux of 0.5 mWb. Determine the magnitude of the core flux produced by a current of 0.8 A in coil 2, with coil 1 open circuited.

Solution: From the definition of mutual inductance, $\frac{N_2\phi_{21}}{i_1} = \frac{N_1\phi_{12}}{i_2}$, so that $\phi_{12} = \frac{i_2}{i_1}\frac{N_2}{N_1}\phi_{21}$ =

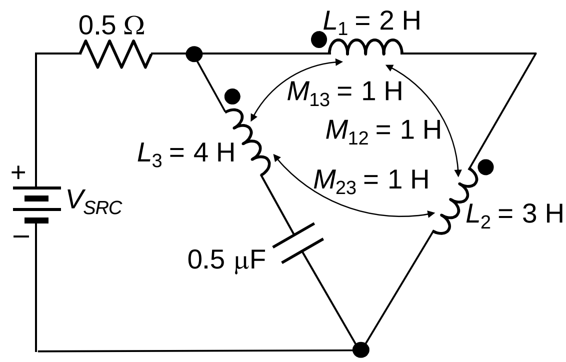
$\frac{0.8}{1}\frac{400}{100}\phi_{21} = 3.2\phi_{21} = 3.2\times 0.5 = 1.6$ mWb

7. Determine $\mathbf{Z_{in}}$, assuming R = 10 Ω, and ω = 1 Mrad/s.



Solution: $M = k\sqrt{L\times L} = 2$ µH. It follows that the series branches of the T-equivalent circuit are zero and the shunt branch is $j\omega M = j2$ Ω; $-\frac{j}{\omega C} = -j$. The parallel impedance of $j2$ and $-j$ Ω is $2/(2-j) = -j2$ Ω. $Z_{in} = R - j2 = 10 - j2$ Ω.

8. Determine the stored **magnetic energy** under dc conditions, assuming V_{SRC} = 1 V.

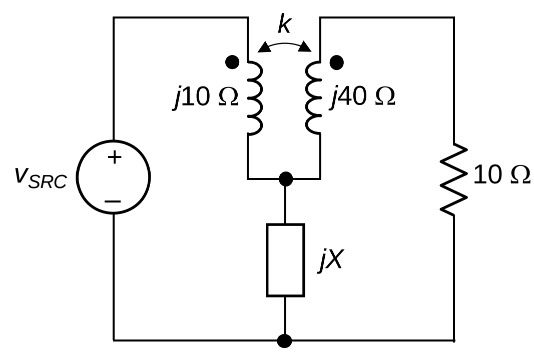


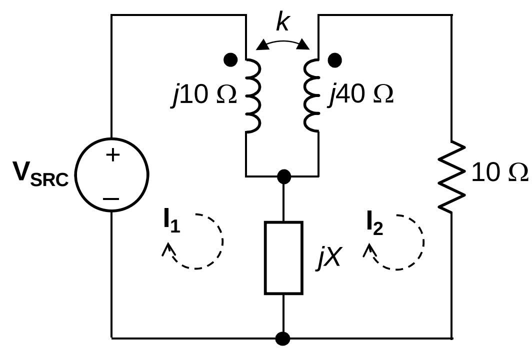
Solution: The branch containing the capacitor carries no current under dc conditions. The current in the other two branches is V_{SRC}/0.5 A and the stored magnetic energy is W =

$$\frac{1}{2}\left(L_1 I^2 + L_1 I^2 + 2MI^2\right) =$$

$$\frac{1}{2}\left(\frac{V_{SRC}}{0.5}\right)^2 (2+3+2\times 1) = 14V_{SRC}^2 = 14 \text{ J.}$$

9. Given $v_{SRC} = 6\cos\omega t$, where ω is unknown and k = 0.97. Determine X so that no power is dissipated in the circuit.





Solution: Considering mesh currents $\mathbf{I_1}$ and $\mathbf{I_2}$, then for no power dissipation, $\mathbf{I_2}$ = 0. The mesh-current equation for mesh 2 is:

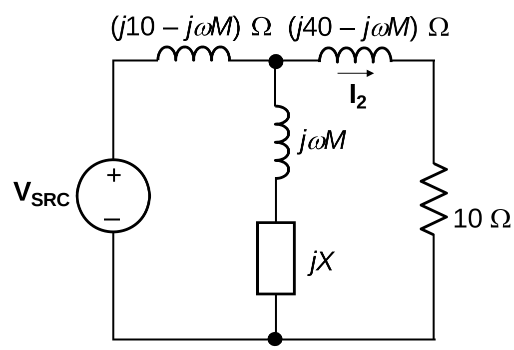
$$-(j\omega M + jX)\mathbf{I_1} + (j40 + jX + 10)\mathbf{I_2} = 0$$

For $\mathbf{I_2}$ to be zero, $\omega M + X = 0$, or $X = -\omega M$ =

$$-\omega k\sqrt{L_1 L_2} = -k\sqrt{(\omega L_1)(\omega L_2)} = -k\sqrt{400} = -20k =$$

$-0.97\times 20 = -19.4$ Ω.

Alternatively, it follows from the T-equivalent circuit that if $X = -\omega M$ the shunt branch will have zero impedance so $\mathbf{I_2}$ = 0.



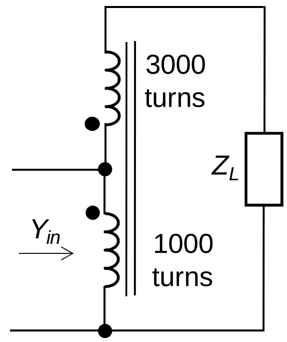
10. Determine the input admittance Y_{in} assuming Z_L = $10\angle 45^\circ\ \Omega$.

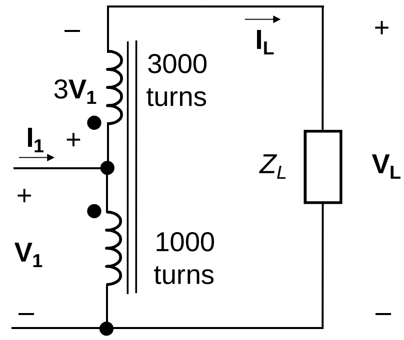
Solution: It follows from the circuit shown that $\mathbf{V_L} = -2\mathbf{V_1}$, so that

$\dfrac{\mathbf{V_L}}{\mathbf{V_1}} = -2$. Since the ideal autotransformer does not dissipate

power, and $\mathbf{V_L} = -2\mathbf{V_1}$, $\mathbf{V_L} \times \mathbf{I_L} = \mathbf{V_1} \times \mathbf{I_1}$, $\dfrac{\mathbf{I_L}}{\mathbf{I_1}} = -\dfrac{1}{2}$. It follows that

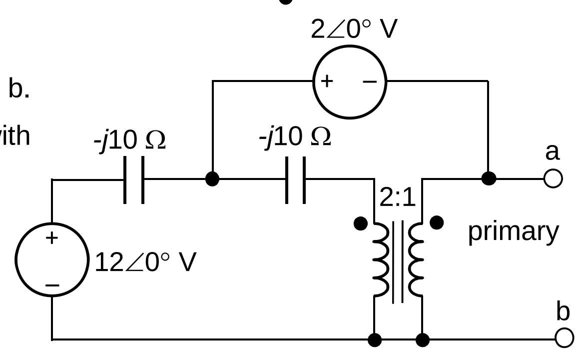
$$Y_{in} = \frac{\mathbf{I_1}}{\mathbf{V_1}} = \frac{4}{Z_L} = \frac{4}{10\angle 45^\circ} = 0.4\angle -45^\circ \text{ S.}$$

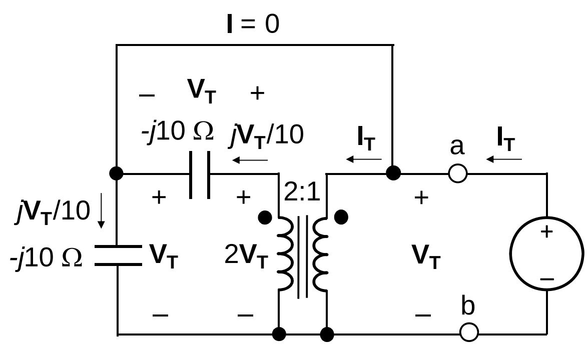




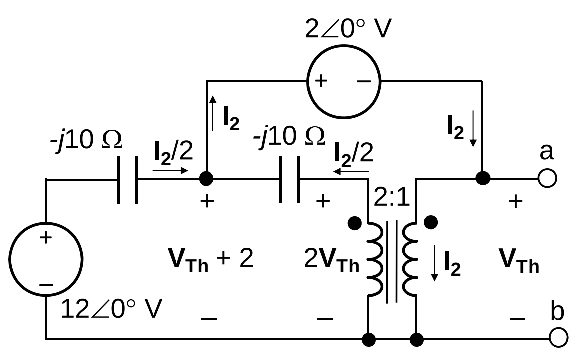
11. Determine Z_{Th} looking into terminals a and b.

Solution-*Method 1*: Apply a test voltage $\mathbf{V_T}$, with the independent sources replaced by short circuits. The secondary voltage is $\mathbf{V_T}$ and the voltage is $2\mathbf{V_T}$. The current through each capacitor is $j\mathbf{V_T}/10$ as shown, so that the current in the short-circuit replacing the 2 V source is zero. For the ideal transformer, $\mathbf{I_T}$ = $j2\mathbf{V_T}/10$. It follows that $\dfrac{\mathbf{V_T}}{\mathbf{I_T}} = \dfrac{5}{j} = -j5\,\Omega$.

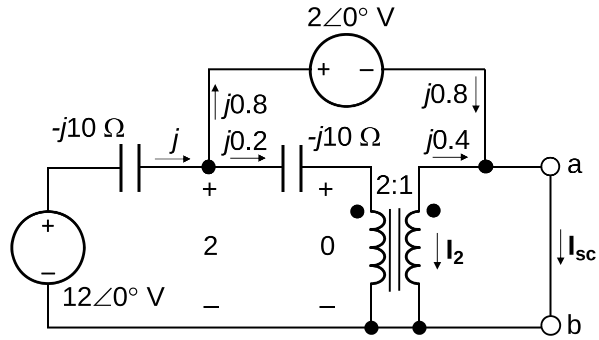




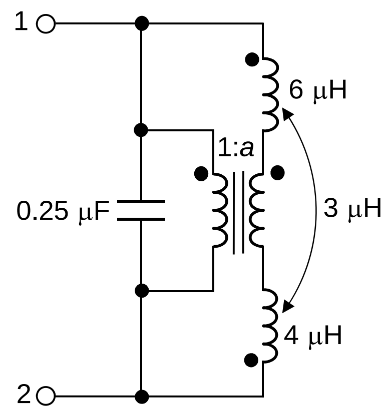
Method 2: Under open-circuit conditions, the currents and voltages will be as shown. The voltages across the two capacitors are of equal magnitude but opposite polarity. It follows that $2\mathbf{V_{Th}}$ = 12 V, and $\mathbf{V_{Th}}$ = 6 V.



Under short circuit conditions, the currents and voltages will be as shown. It follows that $I_{SC} = j1.2$ A. Hence, $Z_{Th} = 6/j1.2 = -j5\ \Omega$.



12. Determine the turns ratio a so that Norton's admittance looking into terminals 1 and 2 is zero, assuming $\omega = 1$ Mrad/s.



Solution-*Method 1:* The impedances are: $j\omega 6 = j6\ \Omega$, $j\omega 4 = j4\ \Omega$, $j\omega 3 = j3\ \Omega$, and $1/j\omega C = 1/(j10^6 \times 0.25 \times 10^{-6}) = -j4\ \Omega$.

When a test source $\mathbf{V_T}$ is applied, the test current $\mathbf{I_T}$ should be zero. The voltages and currents in the circuit will be as shown.

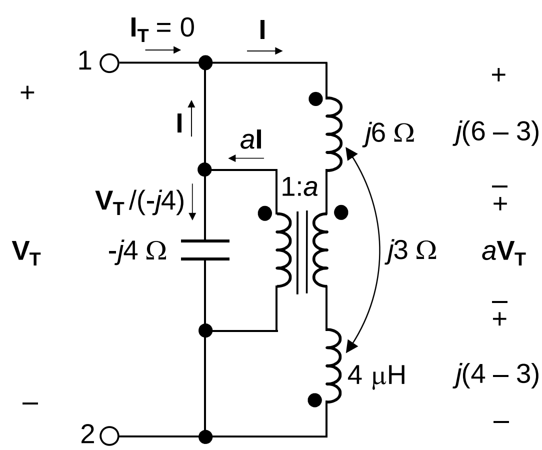
From KVL in the branch containing the coupled coils,

$\mathbf{V_T} = j(6 - 3)\mathbf{I} + a\mathbf{V_T} + j(4 - 3)\mathbf{I} = a\mathbf{V_T} + j4\mathbf{I}$ V,

or $\quad (1 - a)\mathbf{V_T} = j4\mathbf{I} \qquad (1)$

From KCL at node 1, $a\mathbf{I} = \mathbf{I} + \dfrac{\mathbf{V_T}}{-j4}$, or

$$\frac{\mathbf{V_T}}{j4} = (1 - a)\mathbf{I} \qquad (2)$$

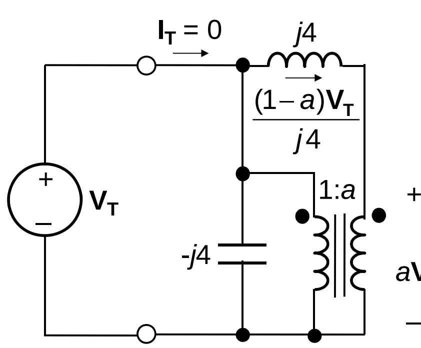


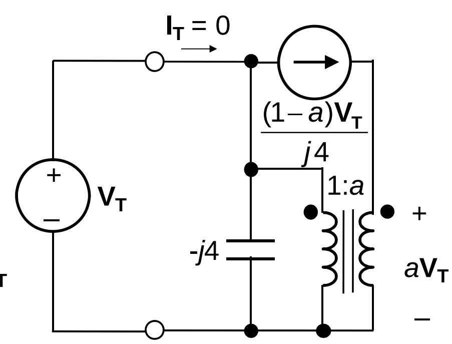
Dividing Equation 1 by Equation 2:

$j4(1 - a) = \dfrac{j4}{(1 - a)}$, $(a - 1)^2 = 1$; $a = 1 \pm 1$,

which gives $a = 2$.

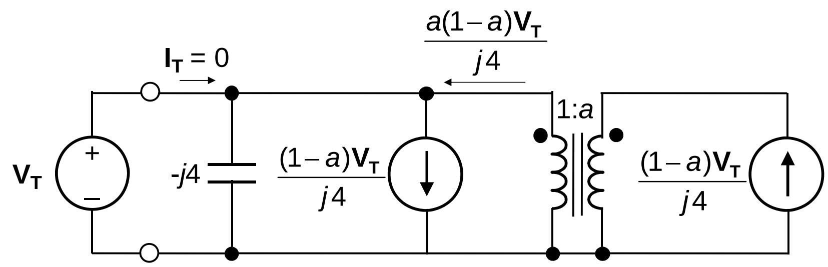
Method 2: The circuit can be redrawn as shown, where the two coupled coils in series have been replaced by the





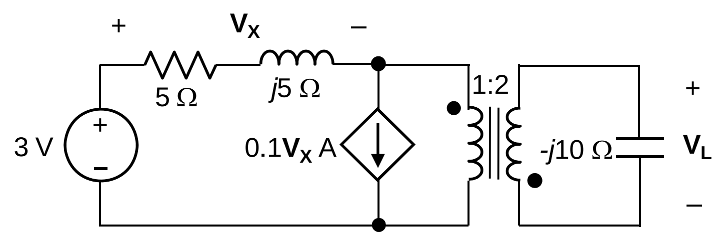
equivalent inductance $(j6 + j4 - j6) = j4\ \Omega$. This inductance carries a current $(1 - a)\mathbf{V_T}/j4$

and can be replaced by a current source of this value in accordance with the substitution theorem. This source can then

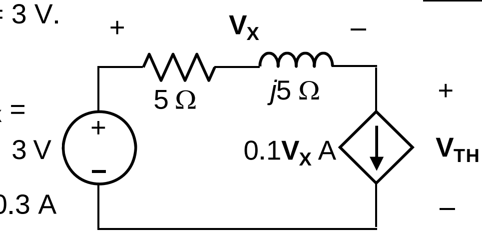


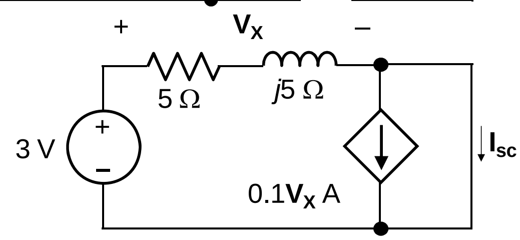
be rearranged as two sources, as shown. From KCL, $\frac{a(1-a)}{j4}\mathbf{V_T} = \frac{(1-a)}{j4}\mathbf{V_T} + \frac{\mathbf{V_T}}{-j4}$, $a - a^2 =$ 1 – a – 1, which gives a = 0 or 2, so a must equal 2.

13. Determine $\mathbf{V_L}$.



Solution-*Method 1*: Derive TEC at the primary terminals. $\mathbf{V_x}$ = 0.1$\mathbf{V_X}$(5 + j5), which gives $\mathbf{V_x}$ = 0 and $\mathbf{V_{TH}}$ = 3 V. On short circuit, $\mathbf{V_X}$ = 3 V and 0.1 $\mathbf{V_X}$ = 0.3 A From KCL.

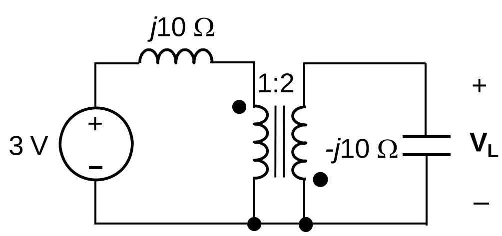


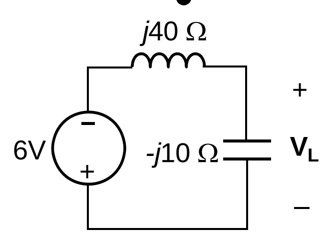


$$\frac{3}{5+j5} = 0.3 + \mathbf{I_{SRC}}$$

which gives, $\mathbf{I_{SRC}}$ = -j0.3 A and Z_{Th} =

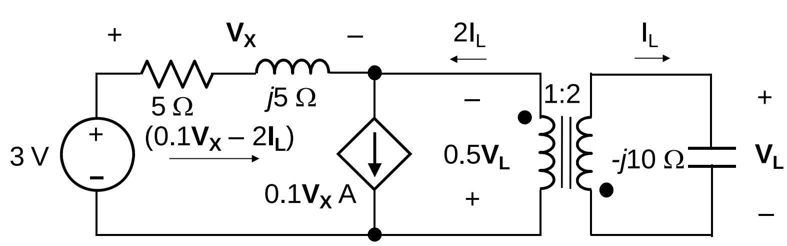
$$\frac{3}{-j0.3} = j10\,\Omega.$$ TEC





becomes as shown. Reflecting this to the secondary, it follows from voltage division that

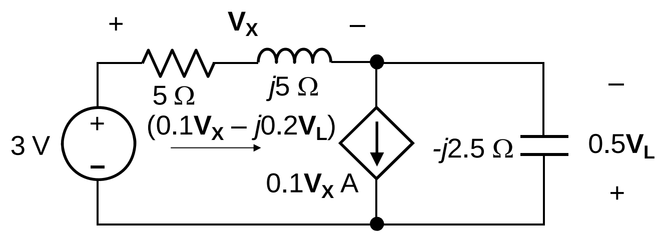
$$\mathbf{V_L} = -\frac{-j10}{j30} \times 6 = 2 \text{ V.}$$



Method 2: No reflections. The primary voltage and current will be as shown in the figure. From KCL, the current in the (5 +j5) Ω impedance is (0.1$\mathbf{V_X}$ – 2$\mathbf{I_L}$), so that $\mathbf{V_X}$ = (0.1$\mathbf{V_X}$ – 2$\mathbf{I_L}$)(5 + j5) and $\mathbf{I_L}$ = $\mathbf{V_L}$/(-j10) = j0.1$\mathbf{V_L}$. Substituting for $\mathbf{I_L}$, $\mathbf{V_X}$ = (0.1$\mathbf{V_X}$ – j0.2$\mathbf{V_L}$)(5 + j5) = 0.5$\mathbf{V_X}$ + j0.5$\mathbf{V_X}$ – j(1 + j)$\mathbf{V_L}$, or, 0.5(1 – j)$\mathbf{V_X}$ = (1 – j)$\mathbf{V_L}$, which gives, $\mathbf{V_X}$ = 2$\mathbf{V_L}$. From KVL on the primary side, 0.5$\mathbf{V_L}$ + 3 – $\mathbf{V_X}$ = 0;

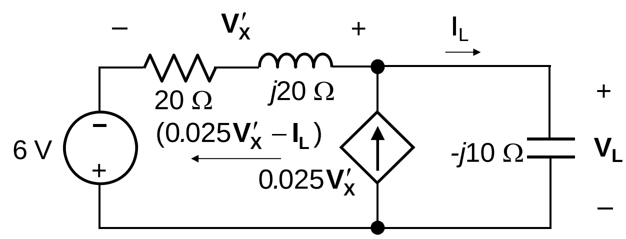
substituting for $\mathbf{V_X}$, $1.5\mathbf{V_L} = 3$ and $\mathbf{V_L} = 2$ V.

Method 3: The capacitive branch is reflected to the primary side together with $\mathbf{V_L}$. From KCL, the current through the $(5 + j5)\ \Omega$



impedance is $0.1\mathbf{V_X} - 0.5\mathbf{V_L}/{-j2.5} = 0.1\mathbf{V_X} - j0.2\mathbf{V_L}$, so that $\mathbf{V_X} = (0.1\mathbf{V_X} - j0.2\mathbf{V_L})(5 + j5) = (0.5\mathbf{V_X} - j\mathbf{V_L})(1 + j) = 0.5\mathbf{V_X} + j0.5\mathbf{V_X} - j\mathbf{V_L}(1 + j)$, or $0.5(1 - j)\mathbf{V_X} = (1 - j)\mathbf{V_L}$, which gives $\mathbf{V_X} = 2\mathbf{V_L}$. From KVL on the primary side, $0.5\mathbf{V_L} + 3 - \mathbf{V_X} = 0$; substituting for $\mathbf{V_X}$, $1.5\mathbf{V_L} = 3$ and $\mathbf{V_L} = 2$ V.

Method 4: The primary circuit is reflected to the secondary side. The $(5 + j5)\ \Omega$ is multiplied by 4. The source voltage and $\mathbf{V'_X}$ are multiplied by 2 and reversed in sign. To maintain the same dependency relation, with the dependent source still



on the primary side, *k* is divided by 2 and reversed in sign. When reflected to the secondary side, this current source must be divided by 2. The overall effect is to divide *k* by 4 and reverse the sign of the current source as shown. It follows that $\mathbf{V'_X} = (0.025\mathbf{V'_X} - \mathbf{I_L})(20 + j20) = (0.5\mathbf{V'_X} - j2\mathbf{V_L})(1 + j) = 0.5\mathbf{V'_X} + j0.5\mathbf{V'_X} + 2\mathbf{V_L}(1 - j)$, or, $0.5\mathbf{V'_X}(1 - j) = 2\mathbf{V_L}(1 - j)$, which gives $\mathbf{V'_X} = 4\mathbf{V_L}$. From KVL, $\mathbf{V_L} - \mathbf{V'_X} + 6 = 0$. Substituting for $\mathbf{V'_X}$, $\mathbf{V_L} = 2$ V.

3. Determine $\mathbf{I_X}$ assuming $\mathbf{I_{SRC}} = j$ A.

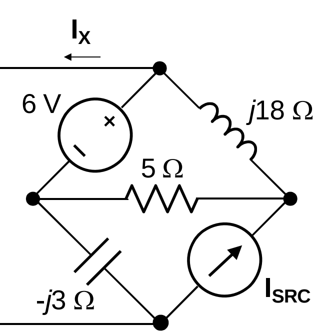
A. $j6$ A

B. $-j3$ A

C. $j3$ A

D. $-j6$ A

E. $j4$ A



Solution: The voltage across the $-j3\ \Omega$ capacitor is 6 V and the current through this capacitor, directed upwards is $j2$ A. It follows that $\mathbf{I_X} = \mathbf{I_{SRC}} + j2 = j3$ A.

5. Two coils are tightly coupled to a high-permeability core, so that the leakage flux is negligibly small. If coil 1 has 100 turns and an inductance of 10 mH, and the mutual inductance is 12.5 mH, determine the number of turns of coil 2.

A. 125
B. 250
C. 150
D. 175
E. 200

Solution: From the definitions of self and mutual inductance, with negligible leakage flux, $L_1 = \frac{N_1\phi_{21}}{i_1}$ and $M = \frac{N_2\phi_{21}}{i_1}$. It follows that $N_2 = \frac{M}{L_1}N_1 = 10M$ = 125.

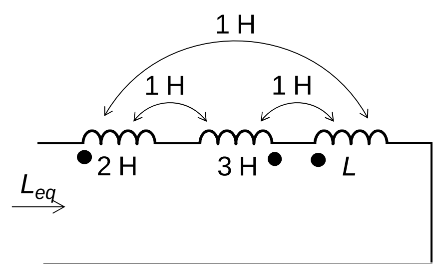
6. Determine the inductance of coil 2 of the preceding problem.

A. 22.5 mH
B. 30.63 mH
C. 15.63 mH
D. 40 mH
E. 50.63 mH

Solution: Since the coils are tightly coupled to the core, $k = 1$, so that $M^2 = L_1L_2$, or $L_2 = \frac{M^2}{L_1} = 0.1M^2$ mH. It also follows from the solution of the preceding problem that $N_1 = \frac{M}{L_2}N_2$. Dividing, $L_2 = L_1\left(\frac{N_2}{N_1}\right)^2 = 0.1M^2 =$ $0.1\times(12.5)^2$ = 15.625 mH.

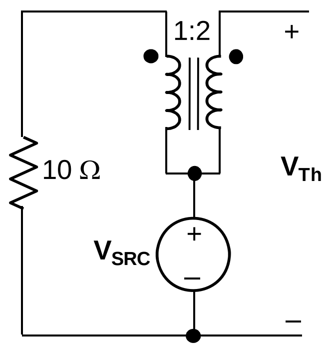
9. Determine L_{eq} if L = 1 H.

A. 6 H
B. 4 H
C. 8 H
D. 7 H
E. 5 H



Solution: Consider that a voltage **V** is applied, causing a current **I** to flow. **V** = $j\omega$**I**[(2 – 1 +1) + (3 – 1 – 1) + (L + 1 – 1)]; L_{eq} = 3 + L = 4 H.

10. Determine $\mathbf{V_{Th}}$, assuming $\mathbf{V_{SRC}} = 1\angle 0° $ V



A. $-1\angle 0°$ V

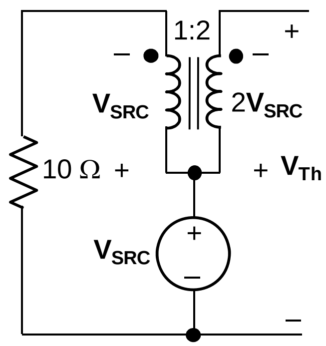
B. $1\angle 0°$ V

C. $-2\angle 0°$ V

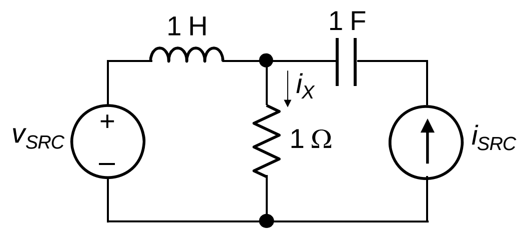
D. $2\angle 0°$ V

E. $4\angle 0°$ V

Solution: On open circuit, no current flows. The primary voltage is $\mathbf{V_{SRC}}$ as shown, and $\mathbf{V_{Th}} = -\mathbf{V_{SRC}} = -1\angle 0°$ V



17. Given $v_{SRC} = \cos t$ V and $i_{SRC} = \sin 2t$ A.



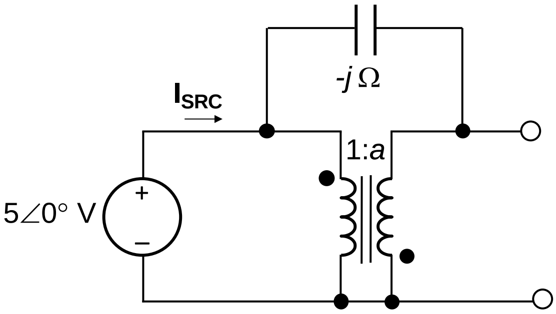
(a) Derive the expression for i_X in the time domain.

(b) Determine the power dissipated in the resistor.

Solution: (a) Let the amplitude of v_{SRC} and i_{SRC} be K. With the current source replaced by an open circuit, $\mathbf{I_{x1}} = \frac{K\angle 0^\circ}{1}\frac{1}{1+j} = \frac{K}{2}(1-j)$; $i_{X1} = \frac{K}{\sqrt{2}}\cos(t-45^\circ)$ A. With the current source replaced by a short circuit, $\mathbf{I_{x2}} = K\angle 0^\circ \frac{j2}{1+j2} = \frac{2K}{\sqrt{5}}\angle(90^\circ - \tan^{-1}2)$; $i_{X2} = \frac{2K}{\sqrt{5}}\sin(2t+26.57^\circ)$ A. Hence, $i_X = \frac{K}{\sqrt{2}}\cos(t-45^\circ) + \frac{2K}{\sqrt{5}}\sin(2t+26.57^\circ) = 0.71\cos(t-45^\circ) +$ $0.89\sin(2t+26.57^\circ)$ A.

(b) Power dissipated is $P = \frac{1}{2}\left(\frac{K^2}{2}+\frac{4K^2}{5}\right) = 0.65K^2 = 0.65$ W.

18. Given the circuit shown, with a = 1.



(a)Determine the current in the capacitor

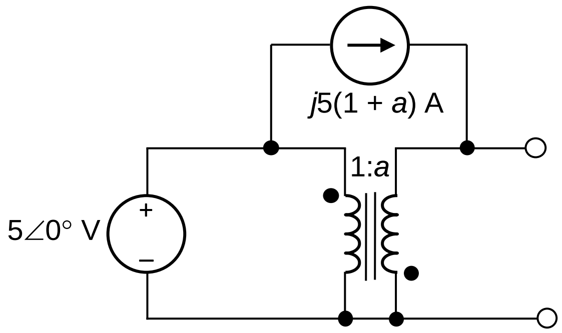
(b)Replace the capacitor by a current source, in accordance with the substitution theorem

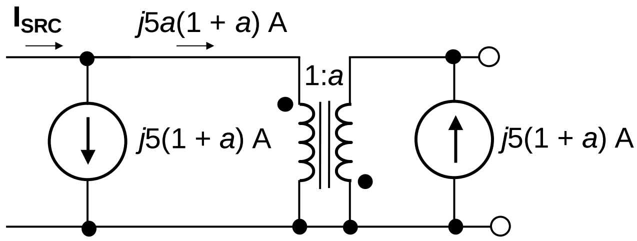
(c)Rearrange the current source as two current sources across the transformer windings

(d)Determine $\mathbf{I}_{\mathbf{SRC}}$.

1. **Solution:** (a) The voltage across the capacitor is 5(1 + a) V. The current through the capacitor is j5(1 + a) A directed from primary to secondary.

(b) (c)



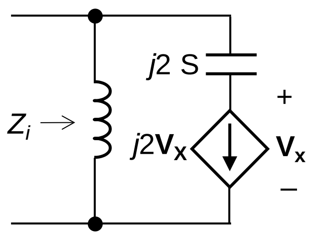


(d) It follows that $\mathbf{I}_{\mathbf{SRC}} = j5(a + 1)^2 = j45$ A.

2. A current $i = I_m\cos(\omega t + 30°)$ flows through an impedance (5 – j5) Ω. Determine the rms phasor voltage across the impedance if I_m = 2.5 A.

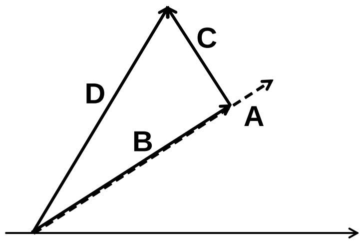
Solution: The phasor current is $I_m\angle 30°$ A. The impedance is $5\sqrt{2}\angle -45^\circ$ Ω. The phasor voltage is $5I_m\sqrt{2}\angle -15^\circ$ V peak value or $5I_m\angle -15^\circ = 12.5\angle -15^\circ$ V rms.

1. Determine Z_i if the susceptance of the inductor is -2 S.



Solution: The dependent current source is equivalent to an admittance of j2 S. In series with j2 S, this becomes j S. In parallel with jB S, $Y_i = jB + j = j(B + 1)$ S, and the impedance is $Z_i = -j/(B + 1)$ Ω.

3. In the phasor diagram shown, phasor **A** is a current, the other phasors **B**, **C**, and **D** are voltages. To which of the following combinations of circuit elements does this phasor diagram apply?



A. *R* in series with *L*

B. *R* in series with *C*

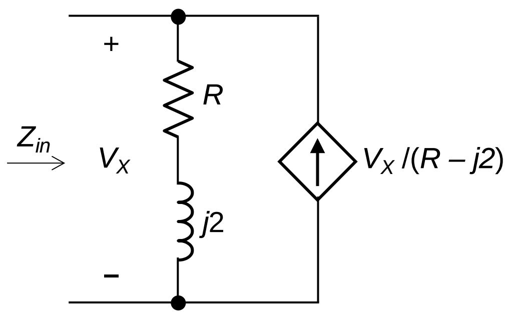
C. *R* in parallel with *C*

D. *L* in series with *C*

E. *L* in parallel with *C.*

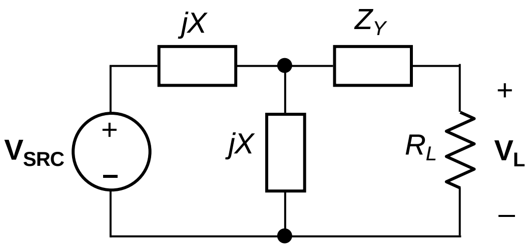
Solution: The phasor diagram represents a series *RL* circuit, Phasor **B** is the voltage across *R*. Phasor **C** is the voltage across *L*, leading the current by 90°, and phasor **D** is the voltage across the series combination.

4. Determine the input impedance Z_{in} assuming $R = 2\ \Omega$.

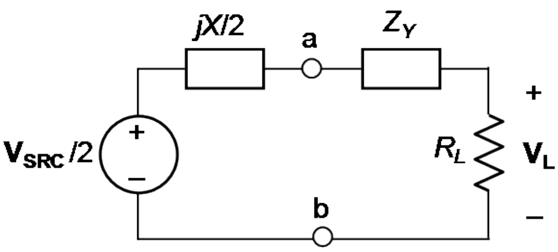


Solution: If a test voltage source $\mathbf{V_T}$ is applied, $\mathbf{I_T} = \left(\frac{1}{R+j2} - \frac{1}{R-j2}\right)\mathbf{V_T}$, or $Z_{in} = \frac{1}{\left(\frac{1}{R+j2} - \frac{1}{R-j2}\right)} = \frac{(R+j2)(R-j2)}{R-j2-R-j2} = \frac{R^2+4}{-j4} = j\left(1+\frac{R^2}{4}\right) = 2\ \Omega$. Alternatively, the dependent source is equivalent to an impedance $-(R - j2)$ and $Z_{in} = (R+j2)\,||\,[-(R-j2)]\Omega$.

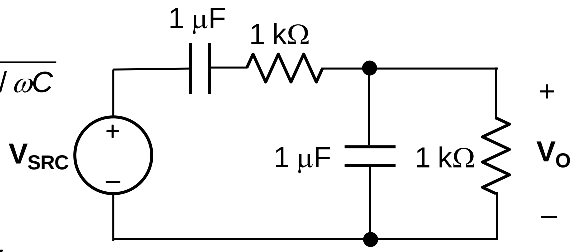
5. Determine Z_Y so that $\mathbf{V_L}$ is in phase with $\mathbf{V_{SRC}}$, assuming $X = -5\ \Omega$ with R_L and $\mathbf{V_{SRC}}$ unknown.



Solution: TEC as seen from terminals a and b will have $Z_{Th} = jX/2$. For $\mathbf{V_L}$ to be in phase with $\mathbf{V_{SRC}}$, $Z_Y = -jX/2$.



6. Determine $\mathbf{V_O}$ if ω = 1 krad/s and $\mathbf{V_{SRC}}$ = 3 V.



Solution: ωCR = 1; $\dfrac{\mathbf{V_O}}{\mathbf{V_{SRC}}} = \dfrac{R/(1+j\omega CR)}{R/(1+j\omega CR)+R-j/\omega C}$

$= \dfrac{R/(1+j)}{R/(1+j)+R-j/\omega C} = \dfrac{1/(1+j)}{1/(1+j)+1-j} =$

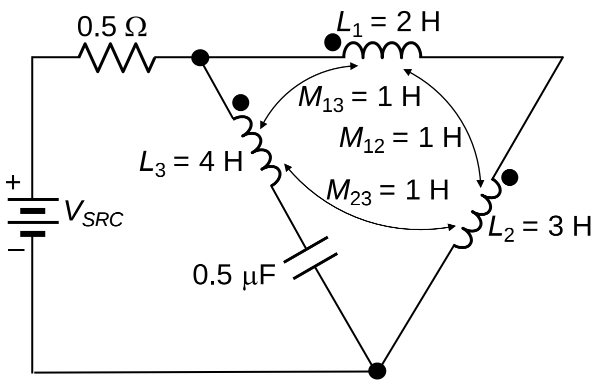
$\dfrac{1}{1+(1+j)(1-j)} = \dfrac{1}{1+1-j^2} = \dfrac{1}{3}$; $\mathbf{V_O} = \mathbf{V_{SRC}}/3$ = 1 V.

7. Two coils are wound on a core of high permeability. Coil 1 has 100 turns and coil 2 has 400 turns. A current of 1 A in coil 1, with coil 2 open circuited, produces a core flux of 0.5 mWb. Determine the magnitude of the core flux produced by a current of 0.8 A in coil 2, with coil 1 open circuited.

Solution: From the definition of mutual inductance, $\dfrac{N_2\phi_{21}}{i_1} = \dfrac{N_1\phi_{12}}{i_2}$, so that $\phi_{12} = \dfrac{i_2}{i_1}\dfrac{N_2}{N_1}\phi_{21}$ =

$\dfrac{0.8}{1}\dfrac{400}{100}\phi_{21} = 3.2\phi_{21}$= 3.2×0.5 = 1.6 mWb.

8. Determine the stored **magnetic energy** under dc conditions, assuming V_{SRC} = 1 V.

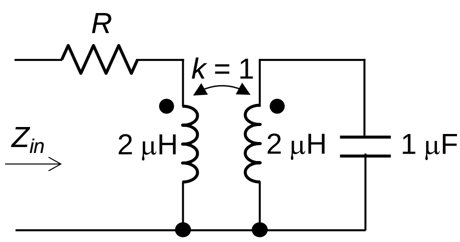


Solution: The branch containing the capacitor carries no current under dc conditions. The current in the other two branches is V_{SRC}/0.5 A and the stored magnetic energy is W =

$\dfrac{1}{2}\left(L_1I^2 + L_1I^2 + 2MI^2\right) =$

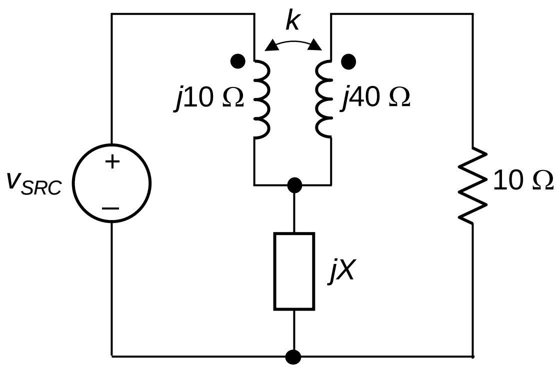
$\dfrac{1}{2}\left(\dfrac{V_{SRC}}{0.5}\right)^2(2+3+2\times 1) = 14V_{SRC}^2$ = 14 J.

9. Determine $\boldsymbol{Z_{in}}$, assuming R = 10 Ω, and ω = 1 Mrad/s.



Solution: $M = k\sqrt{L\times L} = 2$ μH. It follows that the series branches of the T-equivalent circuit are zero and the shunt branch is $j\omega M = j2$ Ω; $-\dfrac{j}{\omega C} = -j$. The parallel impedance of $j2$ and $-j$ Ω is $\dfrac{2}{j2-j} = -j2\,\Omega$. $Z_{in} = R - j2$ = 10 – $j2$ Ω.

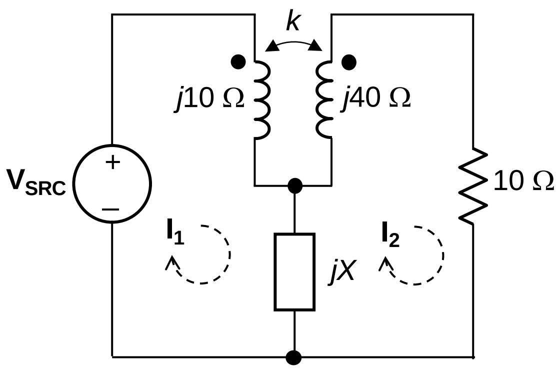
10. Given $v_{SRC} = 6\cos\omega t$, where ω is unknown and $k = 0.97$. Determine X so that no power is dissipated in the circuit.



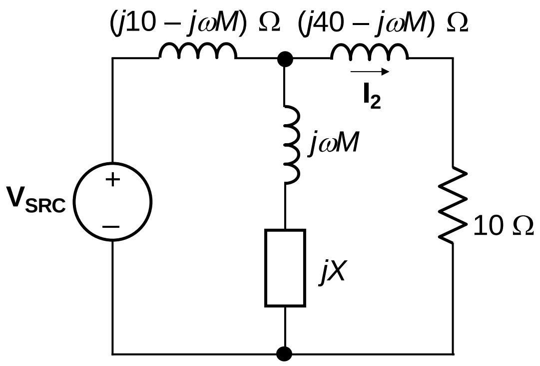
Solution: Considering mesh currents $\mathbf{I_1}$ and $\mathbf{I_2}$, then for no power dissipation, $\mathbf{I_2} = 0$. The mesh-current equation for mesh 2 is:

$$-(j\omega M + jX)\mathbf{I_1} + (j40 + jX + 10)\mathbf{I_2} = 0$$

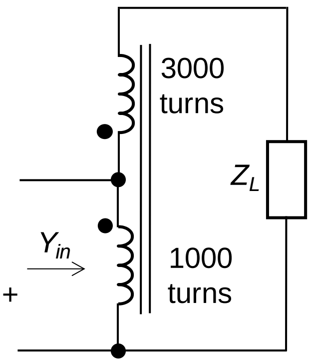
For $\mathbf{I_2}$ to be zero, $\omega M + X = 0$, or $X = -\omega M = -\omega k\sqrt{L_1 L_2} = -k\sqrt{(\omega L_1)(\omega L_2)} = -k\sqrt{400} = -20k = -0.97\times 20 = -19.4\ \Omega$.



Alternatively, it follows from the T-equivalent circuit that if $X = -\omega M$ the shunt branch will have zero impedance so $\mathbf{I_2} = 0$.



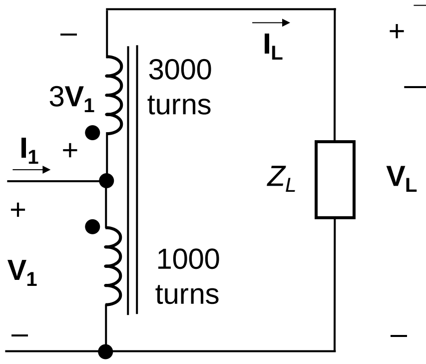
11. Determine the input admittance Y_{in} assuming $Z_L = 10\angle 45^\circ\ \Omega$.



Solution: It follows from the circuit shown that $\mathbf{V_L} = -2\mathbf{V_1}$, so that $\dfrac{\mathbf{V_L}}{\mathbf{V_1}} = -2$. Since the ideal autotransformer does not dissipate power, and $\mathbf{V_L} = -2\mathbf{V_1}$, $\mathbf{V_L}\times\mathbf{I_L} = \mathbf{V_1}\times\mathbf{I_1}$, $\dfrac{\mathbf{I_L}}{\mathbf{I_1}} = -\dfrac{1}{2}$.

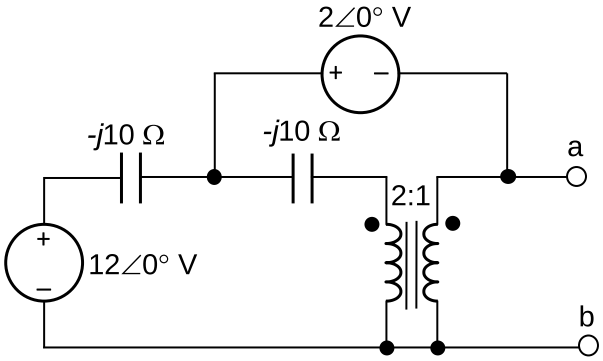
It follows that $Y_{in} = \dfrac{\mathbf{I_1}}{\mathbf{V_1}} = \dfrac{4}{Z_L}$

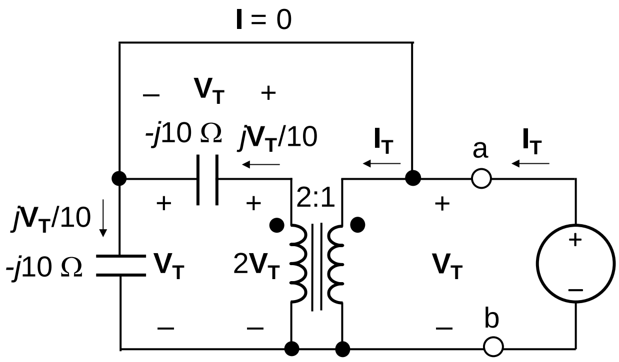
$$= \frac{4}{10\angle 45^\circ} = 0.4\angle -45^\circ\ \text{S}.$$



12. Determine Z_{Th} looking into terminals a and b.

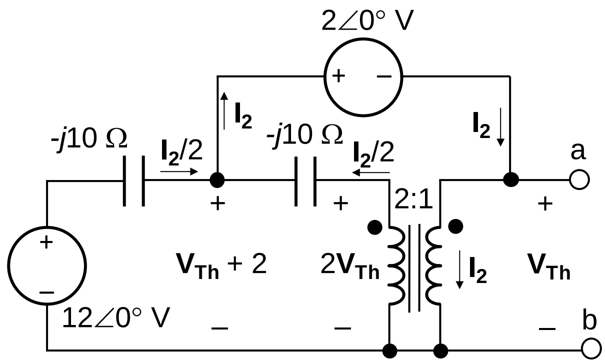
Solution-*Method 1*: Apply a test voltage $\mathbf{V_T}$, with the independent sources replaced by short circuits. The secondary voltage is $\mathbf{V_T}$ and the primary voltage is $2\mathbf{V_T}$. The current through each capacitor is $j\mathbf{V_T}/10$ as shown, so that the current in the short-circuit replacing the 2 V source is zero. For the ideal transformer, $\mathbf{I_T} = j2\mathbf{V_T}/10$. It follows that $\dfrac{\mathbf{V_T}}{\mathbf{I_T}} = \dfrac{5}{j} = -j5\,\Omega$.

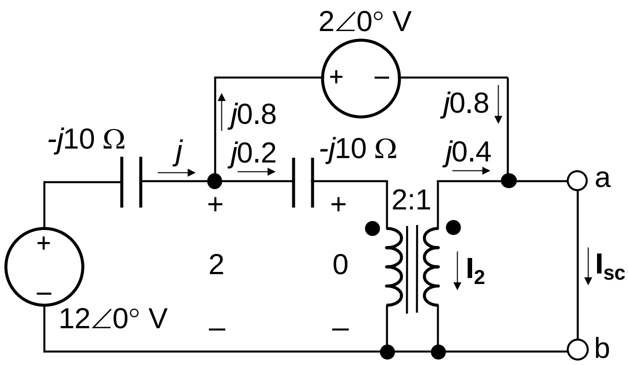




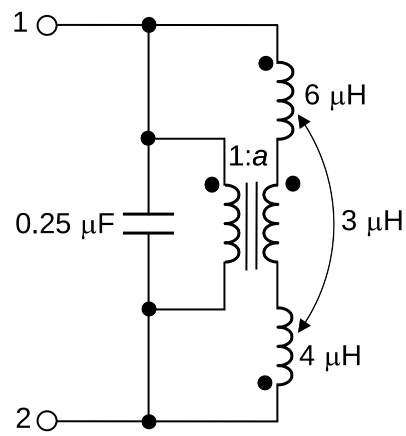
Method 2: Under open-circuit conditions, the currents and voltages will be as shown. The voltages across the two capacitors are of equal magnitude but opposite polarity. It follows that $2\mathbf{V_{Th}} = 12$ V, and $\mathbf{V_{Th}} = 6$ V.

Under short-circuit conditions, the currents and voltages will be as shown. It follows that $I_{SC} = j1.2$ A. Hence, $Z_{Th} = 6/j1.2 = -j5\ \Omega$.





13. Determine the turns ratio a so that Norton's admittance looking into terminals 1 and 2 is zero, assuming $\omega = 1$ Mrad/s.



Solution-*Method 1:* The impedances are: $j\omega 6 = j6\ \Omega$, $j\omega 4 = j4\ \Omega$, $j\omega 3 = j3\ \Omega$, and $1/j\omega C = 1/(j10^6 \times 0.25 \times 10^{-6}) = -j4\ \Omega$.

When a test source $\mathbf{V_T}$ is applied, the test current $\mathbf{I_T}$ should be zero. The voltages and currents in the circuit will be as shown.

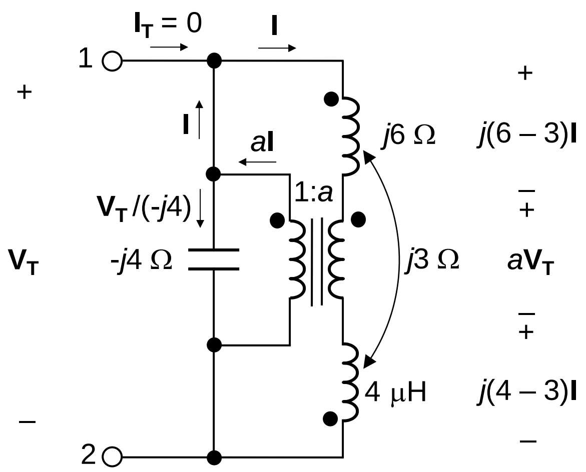
From KVL in the branch containing the coupled coils,

$\mathbf{V_T} = j(6-3)\mathbf{I} + a\mathbf{V_T} + j(4-3)\mathbf{I} = a\mathbf{V_T} + j4\mathbf{I}$ V,

or $\quad (1-a)\mathbf{V_T} = j4\mathbf{I} \qquad (1)$

From KCL at node 1, $a\mathbf{I} = \mathbf{I} + \dfrac{\mathbf{V_T}}{-j4}$, or

$$\frac{\mathbf{V_T}}{j4} = (1-a)\mathbf{I} \qquad (2)$$

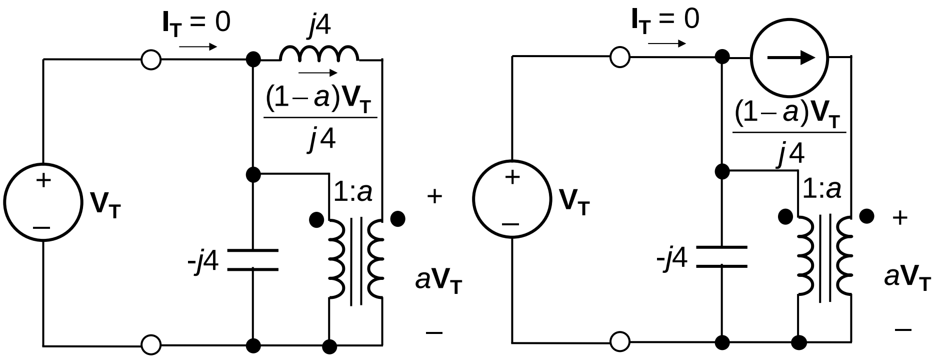


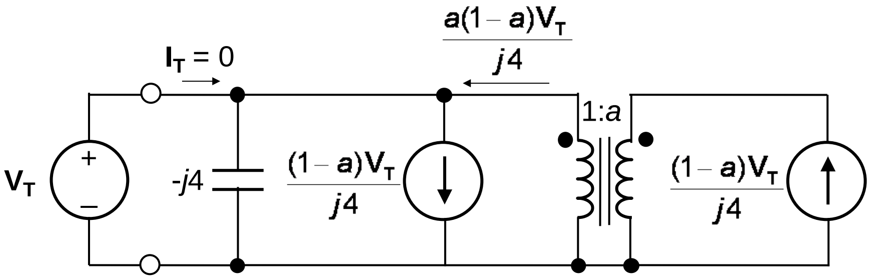
Dividing Equation 1 by Equation 2:

$j4(1-a) = \dfrac{j4}{(1-a)}$, $(a-1)^2 = 1$; $a = 1 \pm 1$,

which gives $a = 2$.

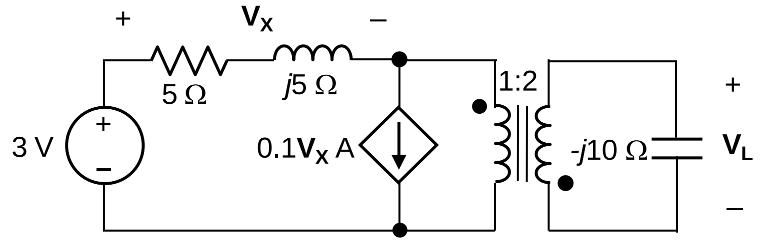
Method 2: The circuit can be redrawn as shown, where the two coupled coils in series have been replaced by the equivalent inductance $(j6 + j4 - j6) = j4\ \Omega$. This inductance carries a current $\dfrac{(1-a)}{j4}\mathbf{V_T}$ and can be replaced by a current source of this value in accordance with the substitution theorem. This source can then be rearranged as two sources, as shown. From KCL,



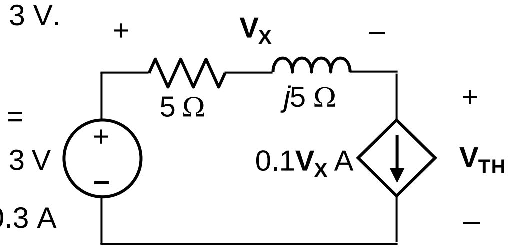


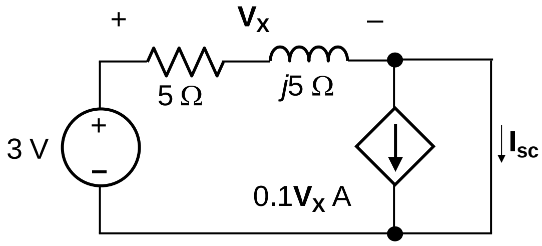
$\dfrac{a(1-a)}{j4}\mathbf{V_T} = \dfrac{(1-a)}{j4}\mathbf{V_T} + \dfrac{\mathbf{V_T}}{-j4}$, $a - a^2 = 1 - a - 1$, which gives $a = 0$ or 2, so a must equal 2.

14. Determine $\mathbf{V_L}$.



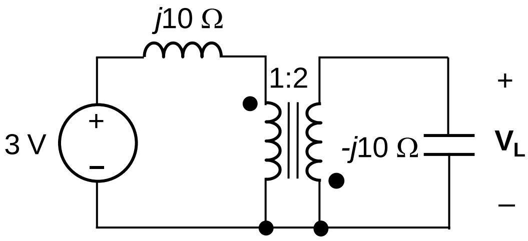
Solution-*Method 1*: Derive TEC at the primary terminals. $\mathbf{V_X} = 0.1\mathbf{V_X}(5 + j5)$, which gives $\mathbf{V_X} = 0$ and $\mathbf{V_{TH}} = 3$ V.

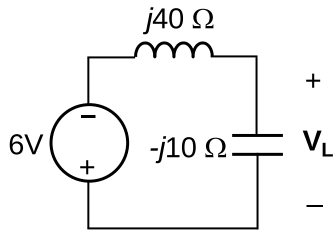




On short circuit, $\mathbf{V_X} = 3$ V and $0.1\,\mathbf{V_X} = 0.3$ A From KCL.

$$\frac{3}{5+j5} = 0.3 + \mathbf{I_{SRC}}$$

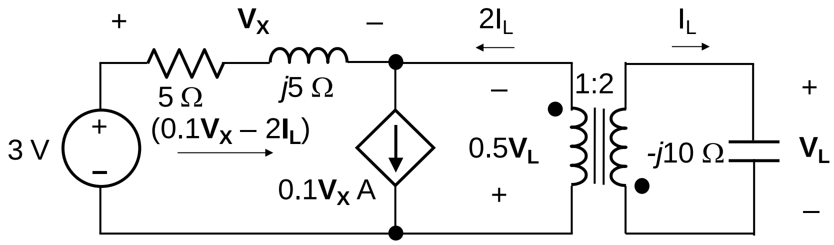




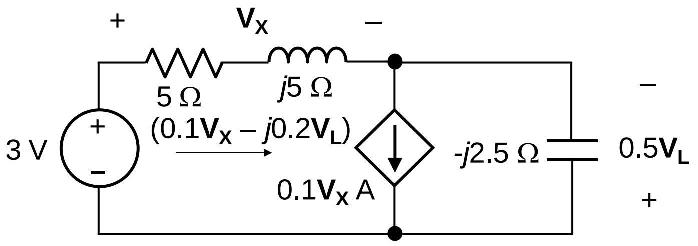
which gives, $\mathbf{I_{SRC}} = -j0.3$ A and $Z_{Th} = \dfrac{3}{-j0.3} = j10\,\Omega$. TEC becomes as shown. Reflecting this to the secondary, it follows from voltage division that

$$\mathbf{V_L} = -\frac{-j10}{j30}\times 6 = 2 \text{ V.}$$

Method 2: No reflections. The primary voltage and current will be as shown in the figure. From KCL, the current in the (5 +j5) Ω impedance is $(0.1\mathbf{V_X} - 2\mathbf{I_L})$,



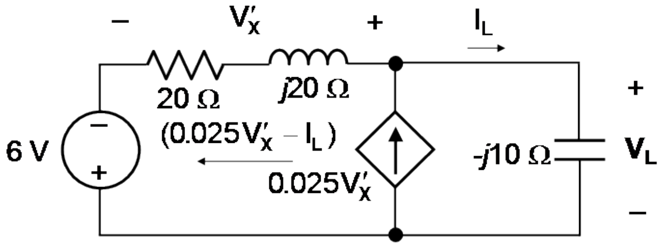
so that $\mathbf{V_X} = (0.1\mathbf{V_X} - 2\mathbf{I_L})(5 + j5)$ and $\mathbf{I_L} = \mathbf{V_L}/(-j10) = j0.1\mathbf{V_L}$. Substituting for $\mathbf{I_L}$, $\mathbf{V_X} = (0.1\mathbf{V_X} - j0.2\mathbf{V_L})(5 + j5) = 0.5\mathbf{V_X} + j0.5\mathbf{V_X} - j(1 + j)\mathbf{V_L}$, or, $0.5(1 - j)\mathbf{V_X} = (1 - j)\mathbf{V_L}$, which gives, $\mathbf{V_X} = 2\mathbf{V_L}$. From KVL on the primary side, $0.5\mathbf{V_L} + 3 - \mathbf{V_X} = 0$; substituting for $\mathbf{V_X}$, $1.5\mathbf{V_L} = 3$ and $\mathbf{V_L} = 2$ V.



Method 3: The capacitive branch is reflected to the primary side together with $\mathbf{V_L}$. From KCL, the current through the (5 +j5) Ω impedance is $0.1\mathbf{V_X} - 0.5\mathbf{V_L}/\text{-}j2.5 = 0.1\mathbf{V_X} - j0.2\mathbf{V_L}$, so that $\mathbf{V_X} = (0.1\mathbf{V_X}$

$- j0.2\mathbf{V}_L)(5 + j5) = (0.5\mathbf{V}_X - j\mathbf{V}_L)(1 + j) = 0.5\mathbf{V}_X + j0.5\mathbf{V}_X - j\mathbf{V}_L(1 + j)$, or $0.5(1 - j)\mathbf{V}_X = (1 - j)\mathbf{V}_L$, which gives $\mathbf{V}_X = 2\mathbf{V}_L$. From KVL on the primary side, $0.5\mathbf{V}_L + 3 - \mathbf{V}_X = 0$; substituting for $\mathbf{V}_X$, $1.5\mathbf{V}_L = 3$ and $\mathbf{V}_L = 2$ V.

Method 4: The primary circuit is reflected to the secondary side. The $(5 + j5)\ \Omega$ is multiplied by 4. The source voltage and $\mathbf{V}'_X$ are multiplied by 2 and reversed in sign. To maintain the same dependency relation, with the dependent source still on the primary side, k is divided by 2 and reversed in sign. When reflected to the secondary side, this current source must be divided by 2. The overall effect is to divide k by 4 and reverse the sign of the current source as shown. It follows that $\mathbf{V}'_X = (0.025\mathbf{V}'_X - \mathbf{I}_L)(20 + j20) = (0.5\mathbf{V}'_X - j2\mathbf{V}_L)(1 + j) = 0.5\mathbf{V}'_X + j0.5\mathbf{V}'_X + 2\mathbf{V}_L(1 - j)$, or, $0.5\mathbf{V}'_X(1 - j) = 2\mathbf{V}_L(1 - j)$, which gives $\mathbf{V}'_X = 4\mathbf{V}_L$. From KVL, $\mathbf{V}_L - \mathbf{V}'_X + 6 = 0$. Substituting for $\mathbf{V}'_X$, $\mathbf{V}_L = 2$ V.



2. Two coils having $N_1 = 1{,}000$ turns and $N_2 = 510$ turns are coupled through a high-permeability core. The inductance of coil 1 is 1 mH and the mutual inductance is 0.5 mH. Determine the ratio of ϕ_{11e} to ϕ_{21}, where ϕ_{11e} is the effective leakage flux of coil 1 and ϕ_{21} is the flux in the core due to current in coil 1.

Solution: $L_1 = \dfrac{\lambda_1}{i_1} = \dfrac{N_1(\phi_{11e} + \phi_{21})}{i_1}$, $M = \dfrac{\lambda_{21}}{i_1} = \dfrac{N_2\phi_{21}}{i_1}$. Dividing, $\dfrac{L_1}{M} = \dfrac{N_1}{N_2}\dfrac{(\phi_{11e} + \phi_{21})}{\phi_{21}}$;

$\dfrac{(\phi_{11e} + \phi_{21})}{\phi_{21}} = \dfrac{L_1}{M} \times \dfrac{N_2}{N_1} = 2 \times \dfrac{N_2}{1000}$; $\dfrac{\phi_{11e}}{\phi_{21}} = \dfrac{N_2}{500} - 1 = \dfrac{510}{500} - 1 = 0.02$. Note that the ratio is zero if $N_2 = 500$. This is because L_2 will then be $\left(\dfrac{500}{1000}\right)^2 \times 1 = 0.25$ mH, and

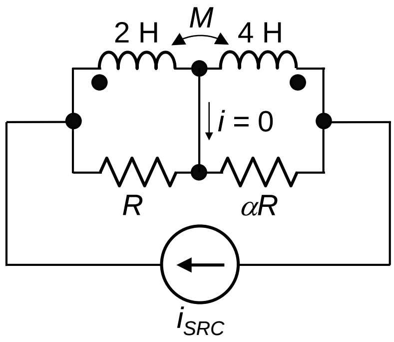
$k = \dfrac{M}{\sqrt{L_1L_2}} = \dfrac{0.5}{\sqrt{1 \times 0.25}} = 1$. The coupling will be perfect, so $\phi_{11e} = 0$.

3. Two identical, magnetically coupled coils are connected in series. When the same current is passed though both coils, but with the connections of one coil reversed, the magnetic stored energy is multiplied by a factor of 2. Determine the coefficient of coupling of the coils.

Solution: The stored energy is $Li^2 \pm Mi^2$. The ratio of the two stored energies is $\dfrac{L + M}{L - M} = \alpha$,

or, $M = \dfrac{\alpha - 1}{\alpha + 1}L$, so $k = \dfrac{\alpha - 1}{\alpha + 1} = \dfrac{\alpha - 1}{\alpha + 1} = \dfrac{1}{3}$.

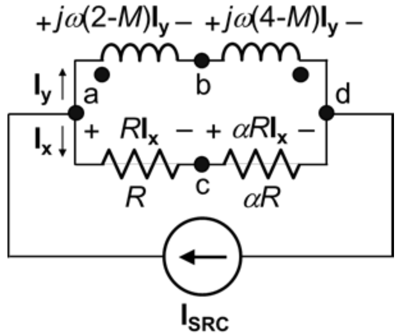
4. In the circuit shown, i_{SRC} and R are unknown, with $\alpha = 3$. Determine M so that $i = 0$.



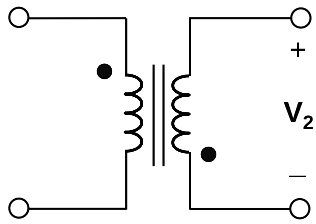
Solution: The circuit in the frequency domain is as shown. When $i = 0$, $\mathbf{V_{bc}} = 0$, and the connection between these nodes can be removed. To have $\mathbf{V_{bc}} = 0$, $\frac{\mathbf{V_{ab}}}{\mathbf{V_{bd}}} = \frac{\mathbf{V_{ac}}}{\mathbf{V_{cd}}}$, or,

$\frac{j\omega(\mathbf{2} - M)\mathbf{I_y}}{j\omega(\mathbf{4} - M)\mathbf{I_y}} = \frac{R\mathbf{I_x}}{\alpha R\mathbf{I_x}}$. This gives; $\alpha(2 - M) = 4 - M$, or,

$$M = \frac{2(\alpha - 2)}{\alpha - 1} = \frac{2 \times 1}{2} = 1.$$



5. In the ideal transformer shown, the phase relation between $\mathbf{V_2}$ and the flux in the core ϕ_c is that:



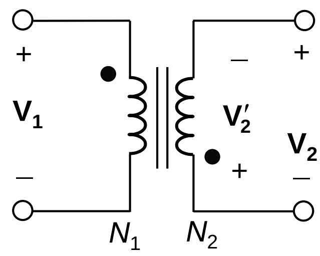
A. $\mathbf{V_2}$ lags ϕ_c by 90°

B. $\mathbf{V_2}$ leads ϕ_c by 90°

C. $\mathbf{V_2}$ lags ϕ_c by 45°

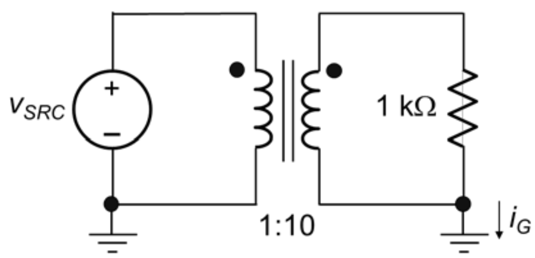
D. $\mathbf{V_2}$ leads ϕ_c by 45°

E. $\mathbf{V_2}$ is in phase with ϕ_c.



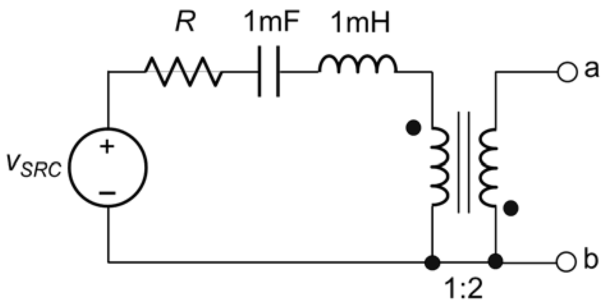
Solution: In phasor notation, $\mathbf{V_1} = j\omega N_1\phi_c$ and $\mathbf{V_2'} = j\omega N_2\phi_c$. Hence, $\mathbf{V_2'}$ leads ϕ_c by 90°, and $\mathbf{V_2} = -\mathbf{V_2'}$ lags ϕ_c by 90°.

6. Given that $v_{SRC} = 10\cos\omega t$ V and the ideal transformer has a turns ratio of 1:10. Determine the current to ground i_G.



Solution: There is no closed path that involves i_G. Hence, $i_G = 0$.

7. Determine the minimum Z_{Th} looking into terminals ab and the frequency at which this minimum occurs, assuming $R = 2\ \Omega$.

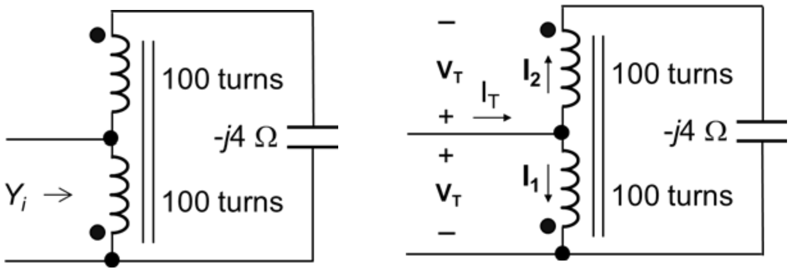


Solution: When the voltage source is set to zero, the impedance looking into terminals ab is $Z_{Th} = \left(\frac{2}{1}\right)^2 \left(R + j\omega L - \frac{j}{\omega C}\right)$. Minimum Z_{Th} is when the imaginary part is zero. This occurs when $\omega L = \frac{1}{\omega C}$, or $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3}10^{-3}}} = 1$ krad/s, and the minimum Z_{Th} is a pure resistance $4R = 8\ \Omega$.

8. Determine $V_{Th} = V_{ab}$ in the preceding problem at a frequency that is twice the frequency for minimum Z_{Th}, assuming $v_{SRC} = 2\cos(\omega t - 30°)$ V.

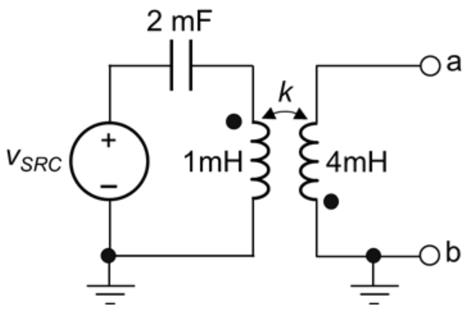
Solution: Since the secondary is open circuited, the primary current is zero at all frequencies. The primary voltage is v_{SRC} and $V_{ab} = -2v_{SRC} = -2V_m\cos(\omega t - 30°) = 2V_m\cos(\omega t + 150°) = 4\cos(\omega t + 150°)$ V.

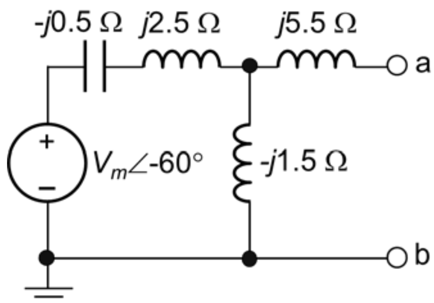
9. Determine Y_i.



Solution: When a test source is applied, the voltage across the capacitor is zero. Hence all currents are zero, and $Y_i = 0$.

10. Determine Thevenin's equivalent circuit looking into terminals ab given that $k = 0.75$ and $v_{SRC} = 2\cos(1{,}000t - 60°)$ V.





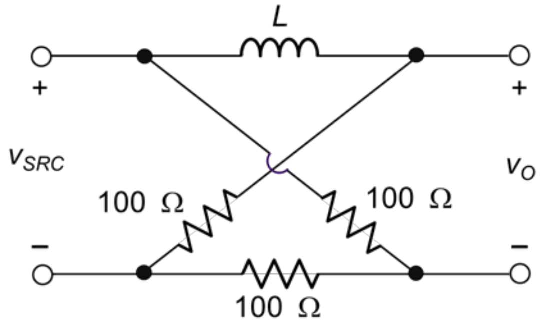
Solution: $1/\omega C = 1/(10^3 \times 2 \times 10^{-3}) = 0.5\ \Omega$; $\omega L_1 = 10^3 \times 10^{-3} = 1\ \Omega$; $\omega L_2 = 10^3 \times 4 \times 10^{-3} = 4\ \Omega$; $\omega M = 10^3 \times 0.75\sqrt{1 \times 4} \times 10^{-3} = 1.5\ \Omega$. Replacing the transformer by its T-equivalent circuit, the circuit becomes as shown, where $j\omega(L_1 + M) = j2.5\ \Omega$ and $j\omega(L_2 + M) = j5.5\ \Omega$. On open circuit,

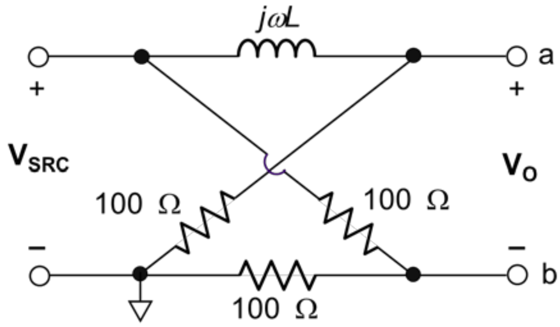
$$\mathbf{V_{Th}} = \frac{-j1.5}{-j0.5 + j2.5 - j1.5} V_m\angle -60^\circ = \frac{-j1.5}{j0.5} V_m\angle -60^\circ = 3V_m\angle 120^\circ = 6\angle 120^\circ \text{ V}.$$

If the source is replaced by a short circuit, the impedance looking into terminals ab is $Z_{Th} =$

$$j5.5 + \frac{-j1.5 \times (-j0.5 + j2.5)}{-j1.5 - j0.5 + j2.5} = j5.5 + \frac{-j1.5 \times j2}{j0.5} = j5.5 - j6 = -j0.5\ \Omega$$

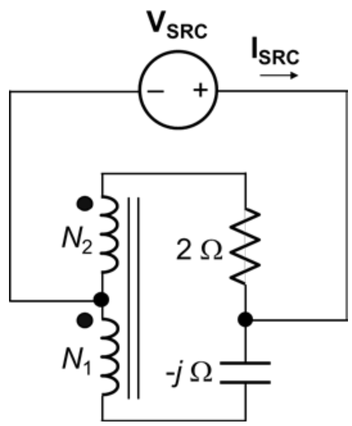
11. Given $v_{SRC} = 10\sin\omega t$ V and $L = 0.1$ H. Determine the frequency at which v_O is 90° out of phase with v_{SRC} and specify whether v_O lags or leads v_{SRC} at this frequency.



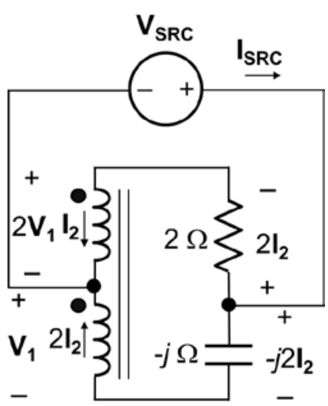


Solution: The circuit in the frequency domain is as shown. The voltages $\mathbf{V_a}$ and $\mathbf{V_b}$ with respect to the reference node are: $\mathbf{V_a} = \dfrac{R}{R + j\omega L}\mathbf{V_{SRC}}$ and $\mathbf{V_b} = \dfrac{\mathbf{V_{SRC}}}{2}$. $\mathbf{V_O} = \mathbf{V_a} - \mathbf{V_b}$ = $\left(\dfrac{R}{R + j\omega L} - \dfrac{1}{2}\right)\mathbf{V_{SRC}} = \dfrac{R - j\omega L}{2(R + j\omega L)}\mathbf{V_{SRC}}$. The phase angle of $\mathbf{V_O}$ with respect to $\mathbf{V_{SRC}}$ is $-2\tan^{-1}\dfrac{\omega L}{R}$, that is lagging. When the magnitude of this angle is 90°, $\tan^{-1}\dfrac{\omega L}{R} = 45^\circ$ and $\dfrac{\omega L}{R} = 1$. This gives $\omega = R/L = 100/L$ = 1000 rad/s.

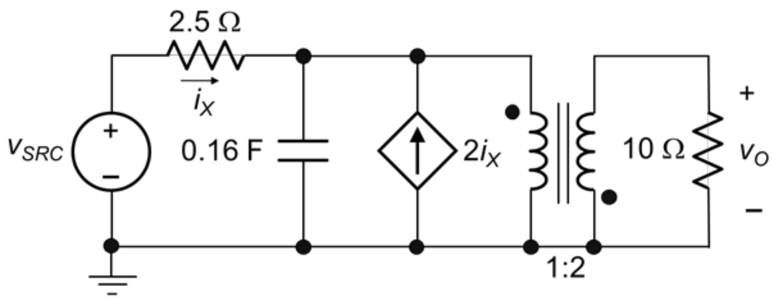
12. Determine $\mathbf{I_{SRC}}$, in polar coordinates, assuming $N_2/N_1 = 2$ and $\mathbf{V_{SRC}} = 1$ V.



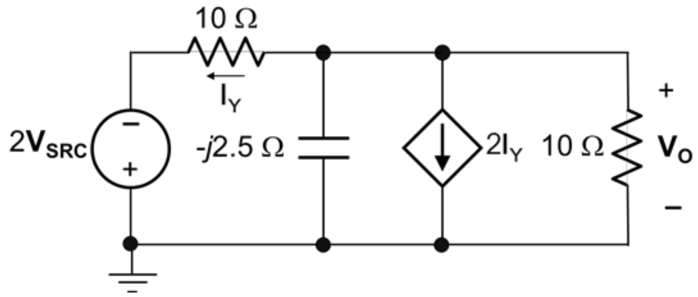
Solution: The voltages and currents of the ideal autotransformer may be assigned as shown. From KVL in the mesh involving R, C, and the transformer: $2\mathbf{I_2} + 2\mathbf{V_1} + \mathbf{V_1} - (-j2\mathbf{I_2}) = 0$, or, $3\mathbf{V_1} = -2(1 + j)\mathbf{I_2}$. From KVL in the upper mesh, $2\mathbf{I_2} + 2\mathbf{V_1} = \mathbf{V_{SRC}}$. Substituting for $\mathbf{V_1}$: $2\mathbf{I_2} - (4/3)(1 + j)\mathbf{I_2} = (2/3)(1 - j2)\mathbf{I_2} = \mathbf{V_{SRC}}$, or, $\mathbf{I_2} = \dfrac{1.5}{1 - j2}\mathbf{V_{SRC}}$. Since $\mathbf{I_{SRC}} = 3\mathbf{I_2}$, $\mathbf{I_{SRC}} = \dfrac{4.5}{1 - j2}\mathbf{V_{SRC}}$ = $0.9(1 + j2)\mathbf{V_{SRC}} = 2.01\angle 63.4^\circ$ A.



13. Determine v_O in the time domain by reflecting the primary circuit to the secondary side, assuming $v_{SRC} = 2\sqrt{2}\cos(10t - 10°)$ V.



Solution: $Z_C = \dfrac{-j}{\omega C} = \dfrac{-j}{1.6}$ $-j\dfrac{5}{8}$ Ω. In reflecting the primary circuit to the secondary side, the 2.5 Ω is multiplied by 4 to become 10 Ω, and Z_C becomes $-j\dfrac{20}{8} = -j2.5\,\Omega$.



$\mathbf{V}_{SRC}$ is multiplied by 2, with sign reversal. Both the current source $2i_X$ and its controlling current i_X are divided by 2, with sign reversal. They become i_X and $0.5i_X$, respectively. They could just as well be designated as $2i_Y$ and i_Y, respectively. The circuit becomes as shown.

From KCL at the upper right-hand node, $\mathbf{V}_O\left(\dfrac{1}{10} + \dfrac{1}{-j2.5}\right) + 3\mathbf{I}_Y = 0$, or

$\mathbf{V}_O(0.1 + j0.4) + 3\mathbf{I}_Y = 0$. From KVL in the outer loop, $\mathbf{I}_Y = \dfrac{\mathbf{V}_O + 2\mathbf{V}_{SRC}}{\mathbf{10}}$, or,

$\mathbf{I}_Y = 0.1\mathbf{V}_O + 0.2\mathbf{V}_{SRC}$. Substituting for $\mathbf{I}_Y$, $\mathbf{V}_O(0.4 + j0.4) + 0.6\mathbf{V}_{SRC} = 0$, or

$\mathbf{V}_O = -\dfrac{1.5\mathbf{V}_{SRC}}{1+j} = -\dfrac{1.5}{\sqrt{2}\angle 45°} = \left(0.75\sqrt{2}\angle 135°\right)\mathbf{V}_{SRC} = 3\angle 125°$, $v_O = 3\cos(10t + 125°)$ V.

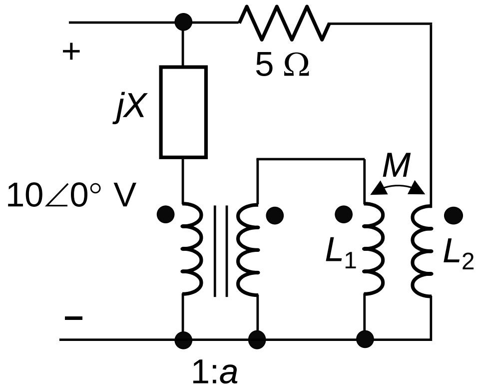
8. Given a load impedance $0.1(4 + j3)$ Ω. Determine the susceptance of the load.

Solution: The load admittance is $Y_L = \dfrac{1}{Z_L} = \dfrac{1}{0.1(4+j3)} = \dfrac{1}{2.5}(4 - j3)$. Hence, $B = -\dfrac{3}{2.5}$ S.

4. If $v_{SRC} = 3\cos\omega t$ V, determine ω at which the voltage across the 10 Ω resistor is a maximum and specify this voltage.

Solution: $v_{SRC} = 3\cos\omega t$ V. The voltage across the 10 Ω resistor is maximum when the parallel impedance of L and C is infinite, that is, when $X_L = -X_C$, which makes $\dfrac{(jX_L)\times(jX_C)}{jX_L + jX_C}$ $\rightarrow \infty$. This occurs when $\omega L = 1/\omega C$, or $\omega = 1/\sqrt{LC} = 1/\sqrt{0.5\times 10^{-3} \times 0.5 \times 10^{-3}} = 2\times 10^3$ rad/s. The voltage is $V_R = 2\times 3/3$ V.

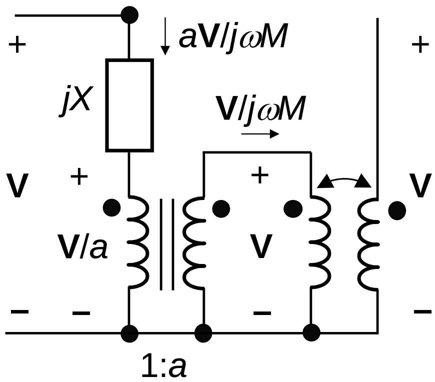
6. Determine X so that the current in the 5 Ω resistor is zero, given that $a = 2$, and for the linear transformer, $\omega L_1 = \omega M = 10\ \Omega$ and $\omega L_2 = 20\ \Omega$.



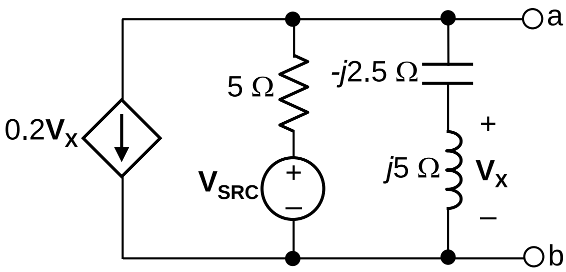
Solution: To have zero current in the resistor, the open-circuited output of the linear transformer should be $\mathbf{V}$. The input current is $\mathbf{V}/j\omega M$, and the input voltage is $\mathbf{V}\frac{j\omega L_1}{j\omega M} = \mathbf{V}$. The primary voltage of the ideal transformer is $\mathbf{V}/a$, and the primary current is $a\mathbf{V}/j\omega M$.

It follows that $jX = \frac{\mathbf{V}(1-1/a)}{a\mathbf{V}/j\omega M} = \left(\frac{1}{a}-\frac{1}{a^2}\right)j\omega M$, or for a = 2,

$$X = \left(\frac{1}{a}-\frac{1}{a^2}\right)\omega M = (0.5-0.25)10 = 2.5\ \Omega$$

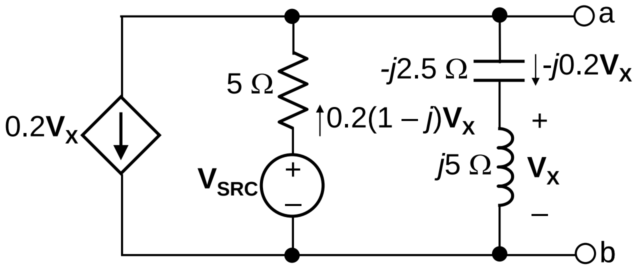


16. Determine Thevenin's equivalent circuit between terminals ab, assuming $\mathbf{V_{SRC}} = 5\angle 0^\circ$ V. Express $\mathbf{V_{Th}}$ in polar coordinates and Z_{Th} in rectangular coordinates.



Solution: The current in the LC branch is $\mathbf{V_X}/j5 = -j0.2\mathbf{V_x}$, and the current in the middle branch is $0.2(1-j)\mathbf{V_x}$. From KVL, $\mathbf{V_{SRC}} - 5\times 0.2(1-j)\mathbf{V_x} = \mathbf{V_X} + (-j2.5)(-j0.2)\mathbf{V_x} = 0.5\mathbf{V_x}$, or $\mathbf{V_x} = \mathbf{V_{SRC}}/(1.5-j)$, $\mathbf{V_{Th}} = 0.5\mathbf{V_X}$

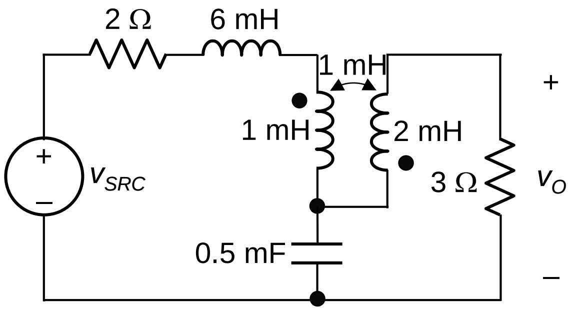
$$= \frac{V_{SRC}}{3-j2} = \frac{V_{SRC}}{13}(3+j2) = \frac{V_{SRC}}{\sqrt{13}}\angle\tan^{-1}(2/3) = \frac{V_{SRC}}{\sqrt{13}}\angle 33.7^\circ \text{ V}.$$



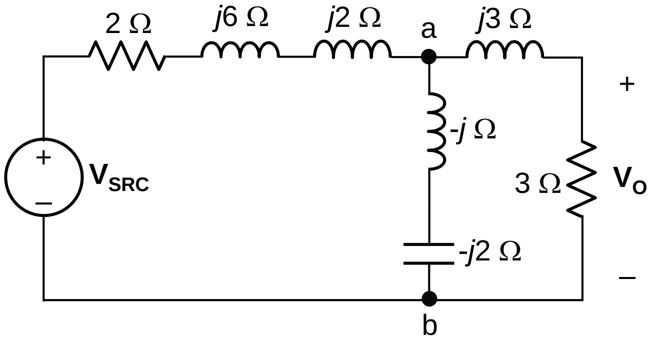
When terminals ab are short circuited, the current source is zero, and $\mathbf{I_{SC}} = \frac{V_{SRC}}{5}$ A. It follows that

$$Z_{Th} = \frac{5}{13}(3+j2) = (1.15+j0.77)\ \Omega.$$

17. Determine $v_O(t)$, assuming v_{SRC} = 2cos1000t V.



Solution: The impedances of the reactive elements are: 6 mH $\rightarrow$ $j6\ \Omega$, 1 mH $\rightarrow$ $j\ \Omega$, 2 mH $\rightarrow$ $j2\ \Omega$, and 0.5 mF $\rightarrow$ $-j2\ \Omega$. The circuit in the frequency domain becomes as shown, with the linear transformer replaced by its T-equivalent circuit. The parallel impedance to the right of terminals ab is $\frac{(3+j3)(-j3)}{3}$ $= 3(1-j)\ \Omega$, and $\mathbf{V_{ab}}$ = $\frac{3(1-j)}{3-j3+2+j8}\mathbf{V_{SRC}} = \frac{3(1-j)}{5(1+j)}\mathbf{V_{SRC}} = -j0.6\mathbf{V_{SRC}}$. It follows that



$$\mathbf{V_O} = \frac{-j0.6\times 3}{3(1+j)}\mathbf{V_{SRC}} = -0.3(1+j)\mathbf{V_{SRC}}$$

$= -0.3\sqrt{2}V_{SRC}\angle 45^\circ$ V, and $v_O(t) = -0.3\sqrt{2}V_m\cos(1000t + 45^\circ)$ = -0.85cos(1000t + 45°) V.